BACHELOR OF SCIENCE (COMPUTER SCIENCE) (CBCS - 2018 COURSE)
S.Y.B.Sc.(Computer Science) Sem-III : WINTER- 2022
SUBJECT : LINEAR ALGEBRA

Day : Saturday

Time : 10:00 AM-01:00 PM

Date : 10/12/2022

W-20093-2022

Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.

Q.1 Attempt **ANY TWO** of the following: **[12]**

a) Let V be a vector space, $\vec{u}$ a vector in V and k be any scalar. Prove that

i) $0 \cdot \vec{u} = \vec{0}$ **ii)** $k \cdot \vec{0} = \vec{0}$

b) Let $V = \mathbb{R}^3$, the vector space of ordered triples of real numbers. If $W = \{(x, y, z) \in \mathbb{R}^3 \mid 2x + 3y + 4z = 0\}$. Show that W is subspace of V.

c) Let V be a vector space with dim $W = n$. If $s = \{\bar{u}_1, \bar{u}_2, ..., \bar{u}_t\}$ is a linearly independent set of vector sin V, and $t < n$, then show that S can be enlarged to a basis for V.

Q.2 Attempt **ANY TWO** of the following: **[12]**

a) Let $T : V \to W$ be a linear transformation, then show that :

i) the Kernel of T is subspace of V.

ii) the range of T is subspace of W.

b) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T(1, 0, 0) = (0, 0)$, $T(0, 1, 0) = (1, 1)$ and $T(0, 0, 1) = (1, -1)$. Compute $T(4, -1, 1)$ and determine the nullity and rank of T.

c) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by

$T(x_1, x_2, x_3) = (-x_1 - x_2 + x_3, x_1 - 4x_2 + x_3, 2x_1 - 5x_2)$

Find the matrix with respect to standard basis B.

Q.3 Attempt **ANY TWO** of the following: **[12]**

a) If A is an $n \times n$ matrix and λ is a real numbers then λ is an eigen value of A if and only if det $(\lambda I - A) = 0$.

b) Find the basis for the eigen spaces of matrix $A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$.

c) Show that the matrix $A = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ is not diagonalizable.

Q.4 Attempt **ANY THREE** of the following: **[12]**

a) Show that $S = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ is linearly independent in $\mathbb{R}^3$.

b) Find the basis for the subspace of $\mathbb{R}^4$ spanned by the vectors $(1, 1, -4, -3)$, $(2, 0, 2, -2)$ and $(2, -1, 3, 2)$.

c) Find the eigen values of matrix $A = \begin{bmatrix} 2 & 7 \\ 1 & -2 \end{bmatrix}$.

d) Let $T : V \to W$ be a linear transformation. Show that, $\ker(T) = \{\bar{0}\}$ if and only if T is one to one map.

Q.5 Attempt **ANY FOUR** of the following: **[12]**

a) Define vector space.

b) Let V be vector space then, show that each vector $\bar{u}$ in V has just one negative in V.

c) Define : **i)** eigen values **ii)** characteristic polynomial.

d) State the rank-nullity theorem for linear transformation.

e) Determine whether a map $T : \mathbb{R}^3 \to \mathbb{R}^3$, defined by $T(x, y, z) = (x + 1, y + 1, z - 1)$ is linear transformation.

f) State Cayley – Hamilton theorem.

* * * *