BACHELOR OF SCIENCE (COMPUTER SCIENCE) (CBCS - 2018 COURSE)
F.Y.B.Sc.(Computer Science) Sem-II : WINTER- 2022
SUBJECT : ALGEBRA-II

Day : Monday Time : 02:00 PM-05:00 PM

Date : 12/12/2022 **W-20081-2022** Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.

Q.1 Attempt any **TWO** of the following: **(12)**

a) Let G be a group. If $a, b \in$ G , then prove that $(ab)^{-1} = b^{-1}a^{-1}$.

b) Let $a, b \in \mathbb{Q}$. Define $a * b = \frac{ab}{2}$. Prove that $*$ is associative commutative and 2 is the identity for $*$ and that every non-zero element of $\mathbb{Q}$ is invertible.

c) Find the order of every element in $(\mathbb{Z}_6, +_6)$.

Q.2 Attempt any **TWO** of the following: **(12)**

a) Prove that every cyclic group is abelian.

b) If H is a subgroup of a group G and $a \in$ G , then prove that Ha = H if and only if $a \in$ H.

c) Find all subgroups of cyclic group $G = \{a, a^2, ...a^{30} = e\}$.

Q.3 Attempt any **TWO** of the following: **(12)**

a) State and prove Lagrange's Theorem.

b) Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 5 & 4 & 3 & 6 & 1 & 7 & 9 & 8 \end{pmatrix}$ as a product of disjoint cycles. Determine σ is even or odd. Find σ^{-1}.

c) Prove that a subgroup H of a group G is normal if and only if $x\text{H}x^{-1} = \text{H}$, for all $x \in G$.

Q.4 Attempt any **THREE** of the following: **(12)**

a) Construct addition table for $(\mathbb{Z}_7, +_7)$.

b) Find the order of each element of the multiplicative group $\{1, -1, i, -i\}$.

c) Prove that intersection of two subgroups is again a subgroup.

d) Show that $A_3 = \{\rho_0, \rho_1, \rho_2\}$ is normal subgroup in S_3.

Q.5 Attempt any **FOUR** of the following: **(12)**

a) Define the term 'Isomorphism'.

b) If G is a group, prove that the identity element in G is unique.

c) Find all subgroups of $(\mathbb{Z}_6, +_6)$

d) Prove that every proper subgroup of a group of order 51 is cyclic.

e) Find $\phi(24)$.

f) Let $\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \in S_3$ then find $|\sigma|$.

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