BACHELOR OF SCIENCE (COMPUTER SCIENCE) (CBCS - 2018 COURSE)
F.Y.B.Sc.(Computer Science) Sem-I : WINTER- 2022
SUBJECT : ALGEBRA-I

Day : Monday | Time : 10:00 AM-01:00 PM

Date : 12/12/2022 | **W-20069-2022** | Max. Marks : 60

N.B.

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable calculator is allowed.

Q.1 Attempt any **TWO** of the following: **(12)**

a) Prove that 8 divides n^2-1 for any odd positive integer n.

b) Find g.c.d. 'd' of 3997 and 2947 are express it in the form $d = 3997\, m + 2947\, n$, for some $m, n, \in \mathbb{Z}$.

c) Let R be define on the set of integers $\mathbb{Z}$ by xRy if $5x+6y$ is divisible by 11, of $x, y \in \mathbb{Z}$. Show that R is an equivalence relation.

Q.2 Attempt any **TWO** of the following: **(12)**

a) Let $f: X \to Y$ and $g: Y \to Z$ be bijective functions. Show that, $(gof): X \to Z$ is also bijective.

b) Prove that for any two non-zero integers a and b have unique LCM. $[a, b]$ and also prove that $[a, b] = \dfrac{|ab|}{(a,b)}$.

c) If p is prime and a, b are integers such that $p \mid ab$, then either $p \mid a$ or $p \mid b$.

Q.3 Attempt any **TWO** of the following: **(12)**

a) Let $z_1, z_2 \in C$, then show that :

i) $|z_1 z_2| = |z_1||z_2|$ and $\arg(z_1 z_2) = \arg z_1 + \arg z_2$

ii) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$, $z_2 \neq 0$ and $\arg\left(\dfrac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$.

b) State and prove De Moivre's Theorem.

c) Let $m = 2,\ n = 5$ *and* $H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Determine (2,5) the group code $e_H : B^2 \to B^5$.

P..T.O.

Q.4 Attempt any **THREE** of the following: **(12)**

a) Find the distance between x and y:
i) $x = 10110101$, $y = 11100110$
ii) $x = 11100011$, $y = 11001011$

b) Find modulus and principal argument of $-3+\sqrt{3}i$.

c) Find remainder of 7^{486} when divided by 13.

d) If $a \equiv b$ (mod n), prove that $(a, b) = (b, n)$

Q.5 Attempt any **FOUR** of the following: **(12)**

a) State Fermat's Theorem.

b) Show that each of the two numbers $z = 1 \pm i$, satisfies the equation $z^2 - 2z + 2 = 0$.

c) Relation R defined in the set of all lines L in plane by $xRy \Rightarrow x \parallel y$ equivalence relation.

d) Let X = {1, 2, 3, 4}, A={1,3}, B{1,4}. Verify the De Morgan's law for A and B.

e) State Division Algorithm.

f) Find the weights of the following words in B^6
i) 011000 **ii)** 010101 **iii)** 111110.

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