BACHELOR OF SCIENCE (CBCS-2018 COURSE)
S. Y. B. Sc. Sem-IV : WINTER- 2022
SUBJECT : MATHEMATICS : VECTOR CALCULUS

Day : Thursday — Time : 02:00 PM-05:00 PM

Date : 15-12-2022 — **W-18392-2022** — Max. Marks : 60

N.B.

1) All questions are **COMPULSORY**.
2) Figures to the **RIGHT** indicate **FULL** marks.
3) Use of non-programmable calculator is **ALLOWED**.

Q.1 Attempt **ANY TWO** of the following : **(12)**

a) A differentiable vector function $\bar{u}(t)$ on [a, b] is of constant magnitude if and only if $\bar{u}.\frac{d\bar{u}}{dt}=0$, $\forall t\in[a,b]$.

b) If $\bar{a}=t^2\hat{i}+t\hat{j}+(2t+1)\hat{k}$ and $\bar{b}=(2t-3)\hat{i}+\hat{j}-t\hat{k}$ find :

i) $\frac{d}{dt}\left(\bar{a}.\bar{b}\right)$ ii) $\frac{d}{dt}\left(\bar{a}\times\frac{d\bar{b}}{dt}\right)$ iii) $\frac{d}{dt}\left(\bar{a}+\bar{b}\right)$ at t = 1 .

c) If $\phi=xyz^2$ and $\bar{a}=3x^2y\hat{i}+yz^2\hat{j}+xyz\hat{k}$, find $\frac{\partial^2\left(\phi\bar{a}\right)}{\partial y\,\partial z}$ at (–1, 1,2) .

Q.2 Attempt **ANY TWO** of the following : **(12)**

a) If $\bar{u}$ be a vector point function and ϕ be a scalar point function then prove that $\nabla\times\left(\phi\bar{u}\right)=(\nabla\phi)\times\bar{u}+\phi\left(\nabla\times\bar{u}\right)$.

b) Find the directional derivative of $\phi=xy^2+yz^3$ at the point (2,–1, 1) in the direction of $\hat{i}+2\hat{j}+2\hat{k}$.

c) If $\bar{r}=x\cos y\hat{i}+y\sin y\hat{j}+ae^{my}\hat{k}$, find $\dfrac{\frac{\partial\bar{r}}{\partial x}\times\frac{\partial\bar{r}}{\partial y}}{\left|\frac{\partial\bar{r}}{\partial x}\times\frac{\partial\bar{r}}{\partial y}\right|}$.

Q.3 Attempt **ANY TWO** of the following : **(12)**

a) If $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$, $\left|\bar{r}\right|=r=\sqrt{x^2+y^2+z^2}$, then prove that

i) $\nabla\phi(r)=\phi'(r)\nabla r$.

ii) ∇r is the unit vector $\hat{r}$.

iii) $\nabla\log r=\frac{\bar{r}}{r^2}$.

P.T.O.

b) Evaluate $\oint_C \left(e^{-x}\sin y\,dx + e^{-x}\cos y\,dy\right)$ where C is the rectangle formed by $x=0,\ x=\pi,\ y=0,\ y=\frac{\pi}{2}$.

c) Verify Stoke's theorem for the vector field $\overline{f} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ over the area in the plane z = 0 bounded by $x = 0$, y = 0 and $x^2 + y^2 = 1$.

Q.4 Attempt **ANY THREE** of the following : **(12)**

a) If $\overline{r} = \frac{a}{2}(x+y)\hat{i} + \frac{b}{2}(x-y)\hat{j} + xy\hat{k}$ find : i) $\left[\frac{\partial \overline{r}}{\partial x}, \frac{\partial \overline{r}}{\partial y}, \frac{\partial^2 \overline{r}}{\partial x^2}\right]$ and ii) $\left[\frac{\partial \overline{r}}{\partial x}, \frac{\partial \overline{r}}{\partial y}, \frac{\partial^2 \overline{r}}{\partial x\,\partial y}\right]$.

b) A particle moves along the curve $\overline{r} = (t^3-4t)\hat{i} + (t^2+4t)\hat{j} + (8t^2-3t^3)\hat{k}$. Find the magnitude of the tangential and normal components of its acceleration at time $t=2$.

c) Find the total work done in moving a particle in a force field $\overline{f} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2+1,\ y = 2t^2,\ z = t^3$ from t = 1 to t = 2 .

d) Evaluate $\iint_S \phi\overline{n}\,ds$, where $\phi = \frac{3xyz}{8}$ and S is the surface of the cylinder $x^2+y^2=16$ included in the first octant between z = 0 and z = 5 .

Q.5 Attempt **ANY FOUR** of the following : **(12)**

a) Show that if $\overline{v}(t)$ is differentiable at $t = t_0$ then it is continuous at $t = t_0$.

b) Find the angle between the surfaces x logz = y^2-1, $x^2y = 2-z$ at the point (1,1,1)

c) Whether $\overline{f} = (y+\sin z)\hat{i} + x\hat{j} + (x+\cos z)\hat{k}$ is irrotational vector field?

d) Evaluate $\int_0^2 \overline{f}\times\overline{g}\,dt$ where $\overline{f}(t) = t\hat{i} - t^2\hat{j} + (t-1)\hat{k}$ and $\overline{g}(t) = 2t^2\hat{i} + 6t\hat{k}$.

e) Eliminate $\overline{a}$ and $\overline{b}$ from $\overline{r} = \overline{a}\cos 2t + \overline{b}\sin 2t$ and obtain the differential equation.

f) If $\overline{a} = t^2\hat{i} - t\hat{j} + (2t+1)\hat{k}$ and $\overline{b} = (2t-3)\hat{i} + \hat{j} - t\hat{k}$, find $\frac{d}{dt}\left(\overline{a}.\overline{b}\right)$.

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