BACHELOR OF SCIENCE (CBCS-2018 COURSE)
S. Y. B. Sc. Sem-IV : WINTER- 2022
SUBJECT : MATHEMATICS : COMPLEX VARIABLES

Day : Saturday | Time : 02:00 PM-05:00 PM

Date : 17-12-2022 | **W-18393-2022** | Max. Marks : 60

N.B.

1) All questions are **COMPULSORY**.
2) Figures to the **RIGHT** indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.

Q.1 Attempt **ANY TWO** of the following : **(12)**

a) A function $f(z)=u(x,y)+iv(x,y)$ is continuous at $z_0=x_0+iy_0$ if and only if u (x, y) and v (x, y) are both continuous at (x_0, y_0).

b) Prove that if $\lim_{z\to z_0} f(z)$ exists, then it is unique.

c) Show that for a function $f(z)=e^{\bar{z}}$, $f(z)$ is not analytic for any z .

Q.2 Attempt **ANY TWO** of the following : **(12)**

a) Show that the function $u=\frac{1}{2}\log(x^2+y^2)$ is Harmonic and find its Harmonic conjugate.

b) Verify the Cauchy Gourast theorem for $f(z)=z+2$ taken around the unit circle $|z|=1$.

c) Prove that an analytic function with constant argument is constant.

Q.3 Attempt **ANY TWO** of the following : **(12)**

a) Prove that if $f(z)$ has a simple pole at $z=z_0$ then the residue of $f(z)$ at $z=z_0$ is $\lim_{z\to z_0}(z-z_0)f(z)$.

b) Evaluate by contour integration $\int_C \frac{3z^2+2}{(z-1)(z^2+9)}dz$ where C is a circle $|z|=4$.

c) Find the value of integral $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}dx$ by using residue .

Q.4 Attempt **ANY THREE** of the following : **(12)**

a) Determine the point of discontinuities of the function $f(z)=\frac{2z-3}{z^2-2z+2}$.

b) If both $f(z)$ and $\overline{f(z)}$ are analytic functions of z then prove that $f(z)$ is constant.

c) Evaluate $\int_C \frac{e^z}{(z^2+1)^2}dz$, where C is the circle $|z-1|=3$.

d) Find the residue of $f(z)=\frac{z^2}{(z-1)(z-2)(z-3)}$ at its simple pole.

P.T.O.

Q.5 Attempt **ANY FOUR** of the following : **(12)**

a) Evaluate $\lim_{z \to i} \frac{iz^3 - 1}{z + i}$.

b) State Cauchy-Gourast Theorem.

c) Evaluate $\int_C \frac{z+6}{z^2-4} dz$, where C is the circle | z | = 1 .

d) Determine the poles and their orders for the function $f(z) = \frac{1}{(z-5)^3 (z-4)^2}$.

e) Discuss the continuity of the function $f(z)$ at $z = 2i$, if

$$f(z) = \frac{z^2 + 4}{z - 2i} \text{ , } if \ z \neq 2i$$
$$= 3 + 4i \text{ , } if \ z = 2i$$

.

f) Prove that $\lim_{z \to 0} \frac{\bar{z}}{z}$ does not exists.

*****