BACHELOR OF SCIENCE (CBCS-2018 COURSE)
S. Y. B. Sc. Sem-III : WINTER- 2022
SUBJECT : MATHEMATICS : CALCULUS OF SEVERAL VARIABLES

Day : Saturday Time : 10:00 AM-01:00 PM

Date : 17-12-2022 **W-18362-2022** Max. Marks : 60

N.B.

1) All questions are **COMPULSORY**.
2) Figures to the **RIGHT** indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following. **(12)**

a) Show that if f be a real-valued function defined on a neighbourhood of (a,b) and f is differentiable at (a,b) then,

i) f is continuous at (a,b)

ii) f_x (a,b) and f_y (a,b) both exist.

b) If $u = \log(x^2 + y^2)$ then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

c) If $u = f(x,y)$ and $x = r\cos\theta,\ y = r\sin\theta$ then show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2 .$$

Q.2 Attempt **ANY TWO** of the following. **(12)**

a) State and prove Taylor's theorem for a function of two variables x and y.

b) Expand $f(x) = x^2y+3y-2$ in powers of $(x-1)$ and $(y+2)$.

c) If $f(x,y) = \frac{x^3y}{x^2+y^2}$, $x^2+y^2 \neq 0$

and $f(0, 0) = 0$ then show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$.

Q.3 Attempt **ANY TWO** of the following. **(12)**

a) Explain Lagrange's method of undetermined multipliers.

b) Investigate the maximum and minimum values of $f(x,y) = (x+y-1)(x^2+y^2)$.

c) Show that the greatest value of 8xyz under the condition $\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{4}=1$ is $\frac{64}{\sqrt{3}}$.

Q.4 Attempt **ANY THREE** of the following. **(12)**

a) Evaluate $\iint xy(x+y)\,dx\,dy$ over the area between $y = x^2$ and $y = x$.

b) Find the volume of sphere of radius a, using spherical polar co-ordinates.

c) Find the area of the region bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

d) Evaluate : $\int_0^1\int_0^{1-x}\int_0^{1-x-y} \frac{dx\,dy\,dz}{(1+x+y+z)^3}$.

Q.5 Attempt **ANY FOUR** of the following. **(12)**

a) Evaluate the limit, if it exists $\lim_{(x,y)\to(0,0)} \frac{x^3y^3}{x^2+y^2}$, $(x^2+y^2 \neq 0)$.

b) If $f(x,y) = 2x^3+3xy^2$ then find f_{xx} and f_{yy} at the point (1, 2).

c) Define: i) Maximum value

ii) Minimum value.

d) Change the order of integration in $\int_0^a\left[\int_0^{\sqrt{a^2-x^2}} f\,dy\right]dx$.

e) Evaluate $\iint_D \frac{x^2}{1+y^2}\,dx\,dy$ where D is the rectangle $0 \le x \le 1,\ 0 \le y \le 1$.

f) Show that the function $f(x,y)$ defined by $f(x,y) = \frac{x^2y^2}{x^2+y^2}$, $(x,y) \neq (0,0)$

$f(0,0) = 0$

is continuous at (0, 0).

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