Day : Tuesday

Time : 10:00 AM-01:00 PM

Date : 20-12-2022   **W-18363-2022**   Max. Marks : 60

N.B.

1) All questions are compulsory.

2) Figures to the right indicate full marks.

Q.1   Attempt **ANY TWO** of the following: [12]

a)   Let $S = \{1, -1, i, -i\}$ and ` * ` is a usual multiplication of complex numbers then show that $(S,*)$ is an abelian group.

b)   Let G be a group. If $a, b \in G$ then prove that $(b^{-1}ab)^m = b^{-1}a^m b$, $\forall \, m \in \mathbb{N}$

c)   Show that intersection of two subgroups of a group is subgroup. Is union of two subgroups of a group is subgroup? Justify.

Q.2   Attempt **ANY TWO** of the following: [12]

a)   Prove that a non-empty set H of a group G is a subgroup of G if and only if $ab^{-1} \in H$, $\forall \, a, b \in H$.

b)   Let G be a group of all non-zero complex numbers $a + ib$, under multiplication. Let $H = \{a + ib / a^2 + b^2 = 1\}$. Show that H is a subgroup of G .

c)   Show that the group $(Z_4 , +_4)$ of residue classes modulo 4 under addition modulo 4 is cyclic. Find all its generators. Find all the proper and improper subgroups.

Q.3   Attempt **ANY TWO** of the following: [12]

a)   Show that for the equation $f(D)y = X$, when $X = e^{ax}$ particular integral is $\frac{1}{f(D)} e^{ax} = \frac{e^{ax}}{f(a)}$, $if \; f(a) \neq 0$.

b)   Solve: $(D^2 + 9)y = x \, sinx$.

c)   Solve: $(D^2 + 4D + 4)y = e^{-2x} + x^3$.

Q.4   Attempt **ANY THREE** of the following: [12]

a)   Solve the differential equation $y = 2px + p^2 y$, where $p = \frac{dy}{dx}$.

b)   Solve : $\left(\frac{dy}{dx}\right)^2 - 5\left(\frac{dy}{dx}\right) + 6 = 0$.

c)   Solve : $y + px = x^4 p^2$, where $p = \frac{dy}{dx}$.

P.T.O.

d) Show that the substitutions $x^2 = u$ and $y^2 = v$ converts equation $(px - y)(py + x) = 2p$ into Clairaut's equation and hence solve it.

Q.5 Attempt **ANY FOUR** of the following: [12]

a) Find the complementary solution of $(D^2 + 2D + 5)y = x \sin2x$.

b) Solve: i) $y = px + \sqrt{a^2p^2 + b^2}$

   ii) $(y - px)^2 = 1 + p^2$

   iii) $y - 2px = f(p^2)$

c) Define: i) Group   ii) Semi-group   iii) Cyclic group

d) Find all the subgroups of a cyclic group of order 24.

e) Prove that in any group every element has unique inverse.

f) Solve: $(D^3 + 7D^2 + 16D + 10)y = 0$.

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