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**BACHELOR OF SCIENCE (CBCS-2018 COURSE)**  
**F. Y. B. Sc. Sem-I : WINTER- 2022**  
**SUBJECT : MATHEMATICS : CALCULUS**

Day : Friday

Time : 10:00 AM-01:00 PM

Date : 16-12-2022

**W-18308-2022**

Max. Marks : 60

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- N.B.:** 1) All questions are **COMPULSORY**.  
2) Figures to the **RIGHT** indicate **FULL** marks.  
3) Use of non-programmable **CALCULATOR** is allowed.
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**Q.1** Attempt **ANY TWO** of the following : **(12)**

- a) Prove that every continuous function on closed and bounded interval attains its bounds.
- b) If a function defined below is continuous in  $[0, 8]$  then find the values of  $a$  and  $b$   
where  $f(x) = x^2 + ax + b$  ,  $0 \leq x < 2$   
 $= 3x + 2$  ,  $2 \leq x \leq 4$   
 $= 2ax + 5b$  ,  $4 < x \leq 8$ .
- c) If  $y = \sin(m \sin^{-1} x)$  then show that  $(1-x^2)y_{n+2} - (2n+1)x y_{n+1} + (m^2 - n^2)y_n = 0$  .

**Q.2** Attempt **ANY TWO** of the following : **(12)**

- a) Show that  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  is divergent if  $p \leq 1$  .
- b) Show that following sequence is monotonic and bounded  
 $0.5, 0.55, 0.555, 0.5555, \dots$
- c) Discuss the convergence of following series by using comparison test  
 $\sum \frac{n+1}{n^3 + 2n^2 + 5}$  .

**Q.3** Attempt **ANY TWO** of the following : **(12)**

- a) State and prove Rolle's mean value theorem.
- b) Find the value of  $c$  and  $\theta$  in the conclusion of L.M.V.T. for the function  
 $f(x) = x^2 - 2x + 3$  over  $[1, 1.5]$  .
- c) Verify Cauchy's mean value theorem for function  $f(x) = \frac{1}{x^2}$  and  $g(x) = \frac{1}{x}$  in  
 $[a, b]$ ,  $a > 0$  . Show that point  $c$  is harmonic mean of  $a$  and  $b$ .

**Q.4** Attempt **ANY THREE** of the following : **(12)**

- a) Evaluate :  $\lim_{x \rightarrow 0} \frac{x \cos x - \log(1+x)}{x^2}$  .
- b) Discuss the continuity of the function  $f(x) = \frac{e^x - 1}{\frac{1}{e^x} + 1}$  where  $x \neq 0$  and  $f(0) = 0$  .
- c) If  $y = \sin(ax + b)$ , then find  $y_n$  .
- d) Prove that  $\log \sec x = \frac{x^2}{2!} + \frac{2x^4}{4!} + \frac{16x^6}{6!} + \dots$

**Q.5** Attempt **ANY FOUR** of the following : **(12)**

- a) Define : 1) removable discontinuity 2) irremovable discontinuity.
- b) Evaluate :  $\lim_{x \rightarrow 0} (\tan x \log x)$  .
- c) Define : i) bounded sequence ii) convergent sequence .
- d) Find the expansion of  $\tan x$  upto the terms in  $x^5$  .
- e) Discuss the continuity of the function  $f(x) = \sqrt{(x-2)(x-4)}$  .
- f) If  $y = e^{ax}$  then find  $y_n$  .

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