BACHELOR OF SCIENCE (CBCS-2018 COURSE)
F. Y. B. Sc. Sem-I : WINTER- 2022
SUBJECT : MATHEMATICS : ALGEBRA

Day : Wednesday | Time : 10:00 AM-01:00 PM

Date : 14-12-2022 | **W-18307-2022** | Max. Marks : 60

N. B. :

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q. 1 Attempt **ANY TWO** of the following: **(12)**

a) Prove that a necessary and sufficient condition for a square matrix A to have the inverse is that A is a non-singular matrix.

b) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

c) Find the non-singular matrices P and Q such that PAQ is the normal form and hence find the rank of matrix A, where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 2 & 3 & 1 \end{bmatrix}.$$

Q. 2 Attempt **ANY TWO** of the following: **(12)**

a) State and prove Division Algorithm (D. A.).

b) Find the g.c.d. of 3587 and 1819 and express the g.c.d. in form 3587 m + 1819 n, for some $m, n \in \mathbb{Z}$.

c) Test the following equations for consistency. If they are consistent find their general solution

$$x - 2y + z - u = 2$$
$$x + y - 2z + 3u = 7$$
$$4x + y - 5z + 8u = 23$$
$$5x - 7y + 2z - u = 15$$

Q. 3 Attempt **ANY TWO** of the following: **(12)**

a) State De Moivre's theorem and prove it for positive and negative integers.

b) Prove that : $\left(1 + i\sqrt{3}\right)^{-10} = 2^{-11}\left(-1 + i\sqrt{3}\right)$.

c) Solve the equation : $x^9 - x^5 + x^4 - 1 = 0$ using De Moivre's theorem.

P. T. O.

Q. 4 Attempt **ANY THREE** of the following: **(12)**

a) Find the eigen values of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$$

b) If $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & x & -1 \\ -1 & x & x \end{bmatrix}$, prove that A^{-1} exists $\forall x \in \mathbb{R}$.

c) If $|z| = 1$ and $\arg z = \theta$ prove that $\frac{1+z}{1-z} = i \cot\left(\frac{\theta}{2}\right)$.

d) If p is prime and a, b are integers such that $p \mid ab$, then prove that either $p \mid a$ or $p \mid b$.

Q. 5 Attempt **ANY FOUR** of the following: **(12)**

a) If $z_1, z_2 \in \mathbb{C}$ then prove that

$$|z_1 z_2| = |z_1|\,|z_2| \text{ and } \arg z_1 z_2 = \arg z_1 + \arg z_2.$$

b) Find modulus and argument of :

$$z = \frac{3-i}{2+i} + \frac{3+i}{2-i}.$$

c) Prove that if $a = b \pmod{n}$ then:

i) $ax \equiv bx \pmod{n}$

ii) $(a + x) \equiv (b + x) \pmod{n}$

for any $a, b, x, \in \mathbb{Z}$.

d) Prove that if $a|b$ and $b|c$ then $a|c$.

e) If $A = \begin{bmatrix} 5 & 4 \\ 1 & -2 \end{bmatrix}$, find A^{-1} by adjoint method.

f) Define:

i) Adjoint of a square matrix

ii) Transpose of a matrix

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