

**BACHELOR OF SCIENCE (CBCS-2018 COURSE)**  
**S. Y. B. Sc. Sem-III :SUMMER- 2022**  
**SUBJECT : MATHEMATICS : GROUP THEORY & DIFFERENTIAL**  
**EQUATIONS**

Day : Saturday  
Date : 16-07-2022

**S-18363-2022**

Time : 03:00 PM-06:00 PM  
Max. Marks : 60

**N.B.:**

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.
- 3) Use of non-programmable **CALCULATOR** is allowed.

**Q.1** Attempt any **TWO** of the following: **(12)**

- a) Prove that in any group  $G$ .
  - i) There is an unique identity element in a group  $G$ .
  - ii) Every element of group  $G$  has an unique inverse.
- b) If  $a, b$  are any two elements in a group  $G$  then show that  $O(a) = O(b^{-1}ab)$ .
- c) Let  $S = \{1, -1, i, -i\}$ . Show that  $(S, \cdot)$  is an abelian group where  $\cdot$  is usual multiplication of complex numbers.

**Q.2** Attempt any **TWO** of the following: **(12)**

- a) Prove that a non-empty subset  $H$  of a group  $G$  is a subgroup of  $G$  if and only if  $ab^{-1} \in H, \forall a, b \in H$ .
- b)  $A$  and  $B$  are subgroups of a group  $G$  such that  $A \cup B$  is also a subgroup of  $G$ . Show that  $A \subseteq B$  or  $B \subseteq A$ .
- c) Show that the group  $(Z_4, +_4)$  of residue classes modulo 4 under addition modulo 4 is cyclic. Find all its generators. Also find all the proper subgroups as well as improper subgroups.

**Q.3** Attempt any **TWO** of the following: **(12)**

- a) Show that if  $f(D)y = e^{ax}V$ , where  $V$  is a function of  $x$  then

$$\frac{1}{f(D)}(e^{ax}V) = \frac{1}{f(D+a)}V.$$

- b) Solve:  $(D^4 + 4)y = \cos 2x + \cos 4x$ .
- c) Solve:  $(D^2 - 1)y = xe^{3x}$ .

**P. T. O.**

**Q.4** Attempt any **THREE** of the following: (12)

a) Show that the substitutions  $x^2 = u$  and  $y^2 = v$  converts equations  $(px - y)(py + x) - 2p$  into Clairaut's equation and hence solve it.

b) Solve:  $p^3 - 4xyp + 8y^2 = 0$ , where  $p = \frac{dy}{dx}$ .

c) Solve:  $\left(\frac{dy}{dx}\right)^2 - 5\left(\frac{dy}{dx}\right) + 6 = 0$

d) Solve:  $(2 + p)x + p^2$ , where  $p = \frac{dy}{dx}$ .

**Q.5** Attempt any **FOUR** of the following: (12)

a) Show that intersection of two subgroups of a group is a subgroup again.

b) Solve:  $(D^6 + 6D^4 + 9D^2)y = 0$ .

c) Solve:

i)  $y - px = \frac{2p}{1 + p^2}$ .

ii)  $y = px + p^3 + 3p^2 + 7$ , where  $p = \frac{dy}{dx}$ .

d) find particular integral of differential equation  $(D^3 - 4D)y = \cos 3x$ .

e) Define: i) Group ii) Abelian group.

f) Find all the subgroups of a cyclic group of order 18.

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