BACHELOR OF SCIENCE (CBCS-2018 COURSE)
S. Y. B. Sc. Sem-III :SUMMER- 2022
SUBJECT : MATHEMATICS : CALCULUS OF SEVERAL VARIABLES

Day : Thursday
Date : 14-07-2022

S-18362-2022

Time : 03:00 PM-06:00 PM
Max. Marks : 60

N.B.

1) All questions are **COMPULSORY**.
2) Figures to the **RIGHT** indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.

Q.1 Attempt **ANY TWO** of the following : **(12)**

a) If a function $f(x, y)$ is differentiable at (a, b) then show that
   i) the partial derivatives $f_x(a, b)$ and $f_y(a, b)$ exist
   ii) f is continuous at (a, b) .

b) If $u=f(x,y)$ and $x=r\cos\theta$, $y=r\sin\theta$ then show that

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2=\left(\frac{\partial u}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2 .$$

c) Prove that $u=\frac{1}{\sqrt{x^2+y^2+z^2}}$ $\left(x^2+y^2+z^2\neq 0\right)$ satisfies the partial differential equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$.

Q.2 Attempt **ANY TWO** of the following : **(12)**

a) State and prove Taylor's theorem for a function of two variables x and y .

b) Expand $f(x, y)=x^3+xy^2$ in power of $(x-2)$ and $(y-1)$.

c) Find an approximate value of $(2.01)\,(3.02)^2$ by using differentials.

Q.3 Attempt **ANY TWO** of the following : **(12)**

a) Explain Lagrange's method of undetermined multipliers.

b) Find maximum value of $\phi(x,y,z)=xyz$ subject to the condition $\frac{x^2}{3}+\frac{y^2}{9}+\frac{z^2}{8}=1$.

c) Using Maclaurin's theorem show that

$$\sin x\sin y=x\,y-\frac{1}{6}\left[\left(x^3+3xy^2\right)\cos\theta x\sin\theta y+(y^3+3x^2y)\sin\theta x\cos\theta y\right],\ 0<\theta<1$$

Q.4 Attempt **ANY THREE** of the following : **(12)**

a) Evaluate $\int_0^1\int_0^{1-x}\int_0^{x+y} e^z\,dx\,dy\,dz$.

b) Evaluate $\iint_D (y-x)\,dx\,dy$ over the region D in the xy-plane bounded by the lines $y=x+1,\ y=x-3,\ y=-\frac{1}{3}x+\frac{7}{3}$ and $y=-\frac{1}{3}x+5$.

c) Four parabolas whose equations are $y^2=4ax,\ y^2=4bx,\ x^2=4cy\ and\ x^2=4dy$ intersect and form a quadrilateral space. Find the area of the space thus enclosed.

d) Change the order of integration and hence evaluate $\int_0^1\left[\int_y^1 e^{-x^2}\,dx\right]dy$.

Q.5 Attempt **ANY FOUR** of the following : **(12)**

a) Show that u is harmonic function if $u=\log\left(x^2+y^2\right)$.

b) If $u=\log(x^3+y^3-x^2y-xy^2)$, then show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=\frac{2}{x+y}$.

c) Define: i) Minimum value
   ii) Extreme value.

d) Evaluate $\iint_R x^2y^3\,dy\,dx$ where R is the rectangle $0\le x\le 1,\ 1\le y\le 3$.

e) Show that $\int_0^1\int_0^1 (x^2+y^2)\,dx\,dy=\frac{2}{3}$.

f) Examine the continuity of $f(x,y)$ at the origin where $f(x,y)=\frac{x+y}{x-y}$ for $x\neq y$ and $f(0,0)=0$.

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