BACHELOR OF TECHNOLOGY (C.B.C.S.) (2020 COURSE)
B.Tech.Sem - III COMPUTER : : SUMMER - 2022
SUBJECT : DISCRETE MATHEMATICS & APPLICATIONS

Day : Monday
Date : 30-05-2022

S-24221-2022

Time : 02:30 PM-05:30 PM
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Draw neat and labeled diagram **WHEREVER** necessary.
4) Assume suitable data if necessary.

Q.1 **a)** Let $P(x)$ be the statement "x spends more than five hours every weekday in class", where the domain for x consists of all students. Express each of these quantification in English **[05]**

i) $\exists \times P(x)$
ii) $\forall \times P(x)$
iii) $\exists \times \neg P(x)$
iv) $\forall \times \neg P(x)$

b) Show that each of these conditional statements is a tautology by using truth tables: **[05]**

i) $(p \wedge q) \rightarrow p$
ii) $p \rightarrow (p \vee q)$
iii) $\neg p \rightarrow (p \rightarrow q)$
iv) $(p \wedge q) \rightarrow (p \rightarrow q)$
v) $\neg (p \rightarrow q) \rightarrow p$

OR

Q.1 **a)** Let A = {a, b, c, d, e} and B = {a, b, c, d, e, f, g, h}. find: **[05]**

i) $A \cup B$
ii) $A \cap B$
iii) $A - B$
iv) $B - A$

b) Show that if A and B are sets, then **[05]**

i) $A - B = A \cap B$
iii) $(A \cap B) \cup (A \cap B) = A$

Q.2 **a)** Determine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set. **[05]**

i) Integers not divisible by 3.
ii) Integers divisible by 5 but not by 7.
iii) The real numbers with decimal representations consisting of all 1s.
iv) The real number.

b) Define the power set of a set S. Define |S|, the cardinality of the set S with suitable example. **[05]**

OR

Q.2 **a)** Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1), (2, 3), (3, 2), (3, 3), (4, 4)\}$. Show that R is equivalence relation and determine equivalence classes. **[05]**

b) Draw Hasse diagram for $X = \{1, 2 \ldots\ldots 7\}$ and $R = \{(x, y) | x - y\}$ is divisible by 3. **[05]**

P.T.O.

Q.3 Determine generating function of the numeric function a_r where: **[10]**

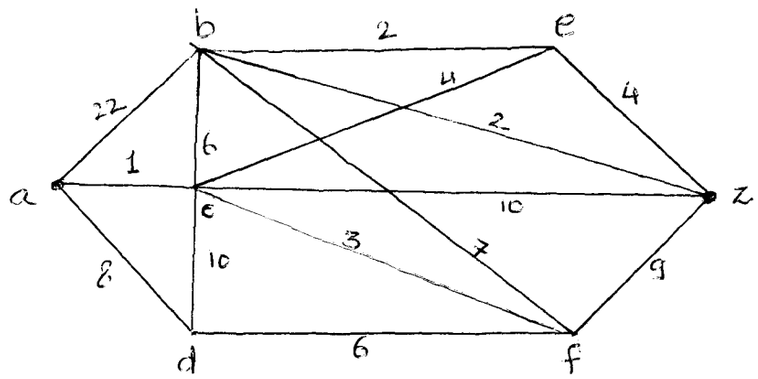
i) $a_r = 3^r + 4^{r+1}, \quad r \geq 0$ **iii)** $a_r = 5, \quad r \geq 0$

OR

Q.3 Determine the discrete numeric functions corresponding to the following generating functions: **[10]**

i) $\dfrac{1}{(1+z)}$ **iii)** $\dfrac{3-5z}{(1-2z-3z^2)}$

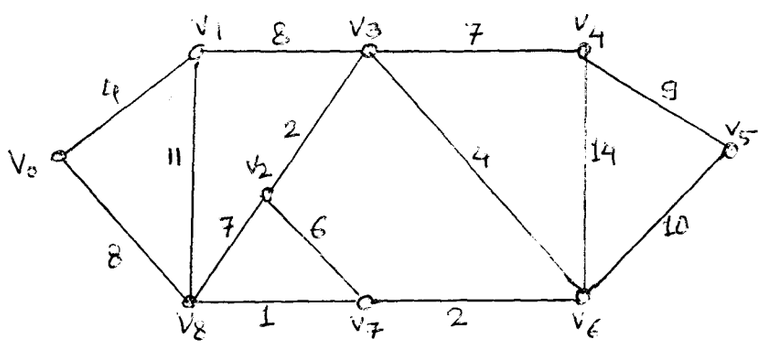
Q.4 Find shortest path between a to z for following graph using Dijkstra's algorithm. **[10]**



OR

Q.4 Determine the number of edges in a graph with 6 nodes 2 of degree 4 and 4 of degree 2. Draw two such graphs. **[10]**

Q.5 Use Prim's algorithm to find minimum spanning tree of following graph: **[10]**



OR

Q.5 Construct an optimal binary tree for the set of weights as {15, 22, 9, 11, 10, 13, 8}. Find weight of an optimal tree. Also assign the prefix codes and write code words. **[10]**

Q.6 What is abelian group? Show that $(Z_6, +)$ is an abelian group. **[10]**

OR

Q.6 Define following with examples: **[10]**

i) Monoid **ii)** Semigroup **iii)** algebraic system

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