BACHELOR OF TECHNOLOGY (C.B.C.S.) (2020 COURSE)
B.Tech.Sem - IV CIVIL :SUMMER- 2022
SUBJECT : VECTOR CALCULUS & DIFFERENTIAL EQUATIONS

Day : Tuesday | Time : 10:00 AM-01:00 PM
Date : 14-06-2022 | **S-24371-2022** | Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Draw neat and labeled diagram wherever necessary.
4) Assume suitable data if necessary.

Q.1 Solve by method of variation of parameters **[10]**
$(D^2-6D+9)\,y = e^{3x}/x^2$.

OR

Q.1 Solve : $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+5y=x^2\sin(\log x)$.. **[10]**

Q.2 A body weight 9.8 is suspended from a spring having constant 4 N/m. Prove that the motion is one of resonance if a force 16 sin2t is applied and damping force is negligible. Assume that initial the weight is at rest in the equilibrium position. **[10]**

OR

Q.2 A horizontal tie-rod is freely pinned at each end. It carries a uniform load w *lb* per unit length and has a horizontal pull P. Find the central deflection and the maximum bending moment, taking the origin at one of its ends. **[10]**

Q.3 A tightly stretched string with fixed and points $x = 0$ and $x = l$ is initially in a position given by $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y at any distance x from one end and at any time t. **[10]**

OR

Q.3 Solve : $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ if *i.*u is finite for all t. **[10]**
ii. u = 0where x = 0, π for all t.
iii. $u=\pi x-x^2$ *when* $t = 0$ *and* $0 \le x \le \pi$

Q.4 Find the directional derivative of the function $\phi = e^{2x-y-z}$ at (1, 1, 1) in the direction of the tangent to the curve **[10]**
$x = e^{-t}$, $y = 2\sin t + 1$, $z = t - \cos t$, at $t = 0$.

OR

Q.4 Show that: **[10]**

a) $\nabla\cdot\left(\frac{\bar{a}\times\bar{r}}{r}\right)=0$

b) $\nabla\times\left(\frac{\bar{a}\times\bar{r}}{r^n}\right)=\frac{(2-n)}{r^n}\bar{a}+\frac{n}{r^{n+2}}(\bar{a}\cdot\bar{r})\,\bar{r}$

P.T.O.

Q.5 Find work done in moving a particle from $(0, 1, -1)$ *to* $\left(\frac{\pi}{2}, -1, 2\right)$ in a force field $\overline{F} = \left(y^2 \cos x + z^3\right)\hat{i} + \left(2y \sin x - 4\right)\hat{j} + \left(3xz^2 + 2\right)\hat{k}$. **[10]**

Is the field conservative?

OR

Q.5 Verify stokes theorem for $\overline{F} = \left(y - z + 2\right)\hat{i} + \left(yz + 4\right)\hat{j} - xz\,\hat{k}$ over the surface of cube $x = 0$, $y = 0$, $z = 0$, $x = 2$, $z = 2$ above the *xoy* plane (open at the bottom). **[10]**

Q.6 Determine the equation of regression lines for the following data: **[10]**

x	1	2	3	4	5	6	7	8	9
y	9	`8	10	12	11	13	14	16	15

And obtain an estimate of y for $x = 4.5$.

OR

Q.6 A manufacturer of cotter pins knows that 2% of his product is defective. If he sells cotter pins in boxes of 100 pins and guarantees that no more than 5 pins will be defective in a box, find the approximate probability that a box will fail to meet the guaranteed quality. **[10]**

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