BACHELOR OF TECHNOLOGY (C.B.C.S.) (2020 COURSE)
B.Tech.Sem - III CS&E : : SUMMER - 2022
SUBJECT : DISCRETE MATHEMATICAL STRUCTURES

Day : Tuesday | Time : 02:30 PM-05:30 PM
Date : 31-05-2022 | **S-24293-2022** | Max. Marks : 60

N.B.

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable calculator is **ALLOWED.**
4) Assume suitable data if necessary.

Q.1 (10)

$P(x)$: x is even
$Q(x)$: x is a prime number
$R(x,y)$: $x + y$ is even
Using above write an English sentence for each of the symbolic statement given below;

i) $\forall x(\sim Q(x))$
ii) $\exists y(\sim P(y))$
iii) $\sim(\exists x(P(x))Q(x)))$
iv) $\exists x(P(x))$
v) $\forall y(\sim Q(x))$

Q.2 (10)

Define various set operations with example and solve the following:
In a survey of 60 people it was found that:
25 read Business India
26 read India Today
26 read Time of India
11 read both Business India and India Today
9 read both Business India and Times of India
8 read both India Today and Times of India
8 read none of these

i) How many read all three?
ii) How many read exactly one?

OR

Define mathematical Induction and prove by induction for
$n \geq 0$

$$1+a+a^2+.....+a^n=\frac{1-a^{n+1}}{1-a}.$$

Q.3 (10)

Let $A = B$ be the set of real numbers
$f : A \to B$ given by $f(x0 = 2x^3 - 1$

$g : B \to A$ given by $g(y) = \sqrt[3]{\frac{1}{2}y+\frac{1}{2}}$.

Show that f is a bijection between A and B and g is bijection between B and A.

OR

$A=\{1,2,3,4,5\}$ and R and S be equivalent relation on A whose matrices are given below. Compute the matrix of smallest relation containing R & S.

$$M_R=\begin{bmatrix}1&1&1&0&0\\1&1&1&0&0\\1&1&1&0&0\\0&0&0&1&1\\0&0&0&1&1\end{bmatrix} \quad M_S=\begin{bmatrix}1&0&0&0&0\\0&1&1&1&0\\0&1&1&1&0\\0&1&1&1&0\\0&0&0&0&1\end{bmatrix}$$

P.T.O.

Q.4 Define: **(10)**

a) Subgroup
b) Cyclic group
c) Integral Domain
d) Field

Prove the following results for the Group G:

i) The identity element is unique.
ii) Each a in G has a unique inverse a^{-1}.
iii) $ab=ac$ implies $b = c$.

OR

Consider a ring $R, +, *)$ defined by $a*a = a$ then determine whether ring is commutative or not.

Define:

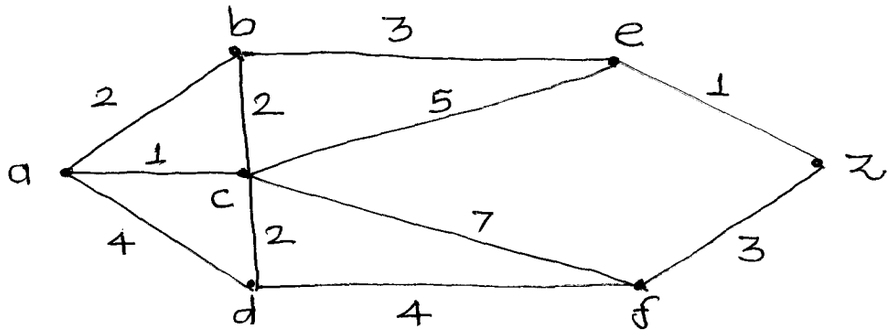
a) Monoid
b) Semigroup
c) Abelian group

Q.5 If repetitions are not permitted, how many four digit numbers can be formed from digits 1, 2, 3, 7, 8 and 5. **(10)**

OR

A committee of 5 people is to be formed from a group of 4 men and 7 women, how many possible committees can be formed if atleast 3 women are on the committee.

Q.6 Use Dijkstra's algorithm to find the shortest path between a & z for given graph. **(10)**

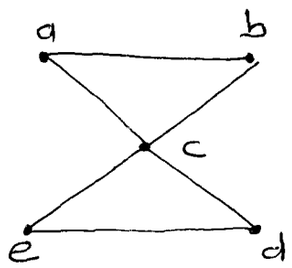


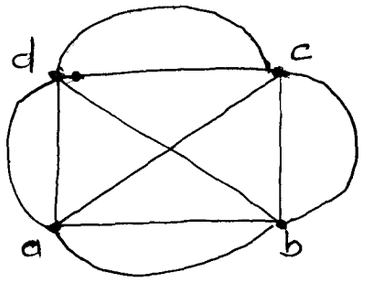
OR

Define :

Euler circuit
Euler path
Hamiltonian path
Hamiltonian circuit

Which of the following graphs have a Euler circuit or path or Hamiltonian cycle? Write the path or circuit.





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