BACHELOR OF TECHNOLOGY (C.B.C.S.) (2021-COURSE)
B. Tech. Sem - II CS&BS :SUMMER- 2022
SUBJECT : LINEAR ALGEBRA

Day : Tuesday
Date : 26-07-2022

S-24136-2022

Time : 10:00 AM-01:00 PM
Max. Marks : 60

N.B.

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable calculator is allowed.

Q.1 Prove that **(10)**

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$$

OR

If

$$\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$$

in which a, b, c are different, show that $abc=1$.

Q.2 Find the rank of matrix A, where **(10)**

$$A = \begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 7 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

OR

Solve the following system of linear equations

$$2x_1 + x_2 - 5x_3 + x_4 = 8$$
$$x_1 + 3x_2 - 6x_4 = -15$$
$$2x_2 - x_3 + 2x_4 = -5$$
$$x_1 + 4x_2 - 7x_3 + 6x_4 = 0$$

Q.3 Find a basis and dimension of the subspace W of P(t) spanned by **(10)**
$U = t^3 + 2t^2 - 2t + 1,\ V = t^3 + 3t^2 - 3t + 4,\ W = 2t^3 + t^2 - 7t - 7$

OR

Let $V = (3, 1-2)$. Find the projection of V on to W, where subspace W spanned by the vectors.
$U_1 = (1,1,1,),\ U_2 = (1,-1,0)$

P.T.O.

Q.4 Apply Gram-Schmidt process to construct an orthonormal basis for the subspace. $W = span(x_1, x_2, x_3)$ of R^4 ,wherte **(10)**

$x_1 = (2, -1, 1, 2)$, $x_2 = (3, -1, 0, 4)$, $x_3 = (1, 1, 1, 1)$

OR

Find a QR-factorization of

$$A = \begin{bmatrix} 2 & 8 & 2 \\ 1 & 7 & -1 \\ -2 & -2 & 1 \end{bmatrix}$$

Q.5 Find eigen values and eigen vectors of matrix A ,where **910)**

$$A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$$

OR

If

$$A = \begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$$

Verify that A*A is a Hermitian Matrix.

Q.6 Find a singular value decomposition of **(10)**

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

OR

Describe the image of unit sphere in R^3 under the action of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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