BACHELOR OF TECHNOLOGY (C.B.C.S.) (2021-COURSE)

B. Tech. Sem - II E&C :SUMMER- 2022

SUBJECT : INTEGRAL TRANSFORMS & VECTOR CALCULUS

Day : Tuesday | Time : 10:00 AM-01:00 PM

Date : 26-07-2022 | **S-24088-2022** | Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.
4) Assume suitable data if necessary.

Q.1 Solve : $\left(y^2e^{xy^2}+4x^3\right)dx+\left(2xy\,e^{xy^2}-3y^2\right)dy=0$. **[10]**

OR

Q.1 Solve : $xy+x^2y^3=\dfrac{dx}{dy}$. **[10]**

Q.2 Show that the differential equation for the current i in an electrical circuit containing an inductance L and resistance R in series and acted on by an electromotive force $E\sin wt$ satisfies the equation $L\dfrac{di}{dt}+Ri=E\sin wt$. Find the value of the current at a time t, if initially there is no current in the circuit.

OR

Q.2 A steam pipe 20 cm in diameter is protected with a covering 6 cm thick for which the coefficient of thermal conductivity is k = 0.0003 *cal/cm.deg.sec* in steady state. Find the heat lost per hour through a meter length of the pipe, if the surface is at 200^0C and the outer surface covering is at 30^0C. **[10]**

Q.3 Solve by method of variation of parameters: **[10]**

$$\frac{d^2y}{dx^2}-y=e^{-x}\sin\left(e^{-x}\right)+\cos\left(e^{-x}\right)$$

OR

Q.3 Solve : $(1+x)^2\dfrac{d^2y}{dx^2}+(1+x)\dfrac{dy}{dx}+y=2\sin\left[\log(1+x)\right]$. **[10]**

Q.4 Show that the Fourier transform of $f(x)=e^{-x^2/2}$ is $e^{-\lambda^2/2}$. **[10]**

OR

P.T.O.

Q.4 Find Z – transform of : **[10]**

$$f(k)=\begin{cases} 2^k & , \quad k<0 \\ \left(\frac{1}{2}\right)^k & , \quad k=0,\ 2,\ 4,\ 6 \\ \left(\frac{1}{3}\right)^k & , \quad k=1,\ 3,\ 5 \end{cases}$$

Q.5 Obtain Laplace transform of : $\frac{e^{-4t}\sin 3t}{t}$. **[10]**

OR

Q.5 Find inverse Laplace transform of : $\frac{2s+5}{s^2+4s+13}$. **[10]**

Q.6 Find the directional derivative of $\phi = e^{2x}\,cosyz$ at (0, 0, 0) in the direction of tangent to the curve $x = asint, y = acost, z = at$ at $t = \pi/4$. **[10]**

OR

Q.6 Find the workdone in a moving a particle once round the ellipse **[10]**

$$\frac{x^2}{25}+\frac{y^2}{16}=1,\ z=0$$

under the field of force given by

$\overline{F}=(2x-y+z)\hat{i}+(x+y-z^2)\hat{j}+(3x-2y+4z)\hat{k}$.

Is the field conservative?

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