BACHELOR OF TECHNOLOGY (C.B.C.S.) (2014 COURSE)
B.Tech.Sem - I CIVIL : : SUMMER - 2022
SUBJECT : ENGINEERING MATHEMATICS-I

Day : Monday
Date : 30-05-2022

S-11247-2022

Time : 10:00 AM-01:00 PM
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of **CALCULATOR** is allowed.

Q.1 **a)** Determine the values of λ for which the following sets of equation possess a non trivial solution and obtain these solution for the s1cal of λ. **(05)**

$$2x-2y+z=\lambda x$$
$$2x-3y+2z=\lambda x$$
$$-x+2y=\lambda z$$

b) Find the eigen values and eigen vectors of the following matrix **(05)**

$$A=\begin{bmatrix}2 & 1 & 1\\ 2 & 3 & 2\\ 3 & 3 & 4\end{bmatrix}.$$

OR

a) Find whether following vectors are linearly dependent or independent and if dependent find the relation between them **(05)**

$x_1=[2,\ 3,\ 4,-2]$; $x_2=[-1,\ -2,\ -2,\ 1]$ and $x_3=[1,\ 1,\ 2,-1]$.

b) Show that the matrix $A=\begin{bmatrix}8 & -8 & -2\\ 4 & -3 & -2\\ 3 & -4 & 1\end{bmatrix}$ satisfies its characteristic equation. **(05)**

Q.2 **a)** Find the locus represented by $|z-3|+|z+3|=10$ **(05)**

b) If $2\cos\left(\frac{\pi}{2^r}\right)=x_r+\frac{1}{x^r}\cos$ **(05)**

Show that $x_1, x_2 ------\infty=-1$.

OR

a) Find in the form $a+ib$ the repression $\cos^{-1}\left(\frac{3i}{4}\right)$. **(05)**

b) Prove that $\tan\left[i\log\frac{a-ib}{a+ib}\right]=\frac{2ab}{a^2-b^2}$. **(05)**

Q.3 **a)** If $x=\sin\theta,\ y=\sin 2\theta,$ prove that **(05)**

$$\left(1-x^2\right)y_{n+2}-\left(2n+1\right)xy_{n+1}-\left(n^2-4\right)y_n=0$$

b) Expand $\sqrt{1+\sin x}$ upto x^4. **(05)**

OR

a) If $I_n=\frac{d^n}{dx^n}\left[x^n log\, x\right]$ Prove that $I_n=nI_n+\left(n-1\right)!$ and hence **(05)**

Show that $I_n=n!\left[\log x+1+\frac{1}{2}+---+\frac{1}{n}\right]$.

P.T.O.

b) Express $2x^3+3x^2-8x+7$ in turn $of\,(x-2)$. **(05)**

Q.4 a) Examine the convenience of the series **(05)**

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+----.$$

b) Find the values of a and b such that $\lim\limits_{x\to\infty}\dfrac{a\sin^2 x+b\cos x}{x^4}=-\dfrac{1}{2}$ **(05)**

OR

Test the convergine of the **(10)**

i) $\dfrac{1}{\sqrt{1}+\sqrt{2}}+\dfrac{1}{\sqrt{2}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+\sqrt{4}}----$

ii) $\dfrac{1}{1.2.3}+\dfrac{3}{2.3.4}+\dfrac{5}{3.4.5}+---$

Q.5 a) If $u=f(r)$ and $x=r\cos\theta, y=r\sin\theta$, prove that **(05)**

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r).$$

b) If $u=\tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\sin 2u$ and **(05)**

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}=2\cos 3u\sin u.$$

OR

a) If $z=2xy^2-3x^2y$ and if x is increasing at the rate of $2cm.$ per second and it passes through the value $x=3cm,$ show that if y is passing through the value $y=1cm, y$ must be decreasing at the rate of $2\frac{2}{15}cm.$ per second in order that z shall remain constant. **(05)**

b) If $x=\sqrt{vw}, y=\sqrt{uw}, z=\sqrt{uv}$, and ϕ is function $of\ x,y,z$. Show that **(05)**

$$x\frac{\partial\phi}{\partial x}+y\frac{\partial\phi}{\partial y}+z\frac{\partial\phi}{\partial z}=u\frac{\partial\phi}{\partial u}+v\frac{\partial\phi}{\partial v}+w\frac{\partial\phi}{\partial w}.$$

Q.6 a) A rectangular box open at the top is to have volume 32ft^3. Find the dimensions of the box requiring least material for its manufacturing. **(05)**

b) Verify $JJ'=1$, for $x=e^v\sec u,\ y=e^v\tan u.$ **(05)**

OR

Find the maximum value of $u=x^4+y^4-2x^2+4xy-2y^2$. **(10)**

* * * *