BACHELOR OF SCIENCE (COMPUTER SCIENCE) (CBCS - 2018 COURSE)
S.Y.B.Sc.(Computer Science) Sem-III :SUMMER- 2022
SUBJECT : LINEAR ALGEBRA

Day : Thursday
Date : 7/7/2022

S-20093-2022

Time : 03:00 PM-06:00 PM
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.

Q.1 Attempt **ANY TWO** of the following: **(12)**

a) Show that intersection of two subspaces of a given vector space is a subspace. Whether union of two subspaces of a given vector space is a subspace?

b) Determine whether the matrices $A=\begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix}, B=\begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}, C=\begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}$ are linearly dependent in $M(\mathbb{R})_{2\times 2}$.

c) Find the dimension and basis for the solution space of the system

$$x_1 + 2x_2 - x_3 + 3x_4 = 0$$
$$2x_1 + 2x_2 - x_3 + 2x_4 = 0$$
$$x_1 + 3x_3 + 3x_4 = 0.$$

Q.2 Attempt **ANY TWO** of the following: **(12)**

a) Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is any 2×2 matrix, then show that if A is invertible, then $ad - bc \neq 0$ and $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

b) Solve the following system of equations by Gauss elimination method,

$$x_1 + 2x_2 + 3x_3 = 0$$
$$2x_1 + 3x_2 + x_3 = 0$$
$$4x_1 + 5x_2 + 4x_3 = 0$$
$$x_1 + x_2 - 2x_3 = 0$$

c) Find an LU factorization of the coefficient matrix,

$$A=\begin{bmatrix} 2 & 3 & 0 & 1 \\ 4 & 5 & 3 & 3 \\ -2 & -6 & 7 & 7 \\ 8 & 9 & 5 & 21 \end{bmatrix}.$$

P.T.O.

Q.3 Attempt **ANY TWO** of the following: **(12)**

a) Prove that if λ is an eigenvalues of a square matrix A, then λ^m is an eigenvalue of A^m for every positive integer m.

b) Find all eigenvalues of a matrix A and hence write eigenvalues of A^{-1} and A^4 ,where $A = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{bmatrix}$.

c) Find a matrix P that diagonalizes matrix $A = \begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$ also write $P^{-1} AP$.

Q.4 Attempt **ANY THREE** of the following: **(12)**

a) Show that if $T: V \to W$ is a linear transformation then ker T is a subspace of V.

b) A mapping $T: \mathbb{R}^2 \to \mathbb{R}^3$ is defined as $T(x, y) = (x+y, x-y, 1)$. Determine whether T is a linear transformation.

c) Find a basis for range T when $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by $T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} a+c \\ a+b \\ b-c \end{bmatrix}$.

d) If $T: V \to W$ be a linear transformation then show that

i) $T(\bar{0}) = \bar{0}$ ii) $T(\bar{v} - \bar{w}) = T(\bar{v}) - T(\bar{w})$.

Q.5 Attempt **ANY FOUR** of the following: **(12)**

a) Find the dot product of $\bar{u} = \begin{bmatrix} 2 \\ -3 \\ 5 \\ 4 \end{bmatrix}$ and $\bar{v} = \begin{bmatrix} 4 \\ 1 \\ -2 \\ 3 \end{bmatrix}$

b) Define a vector space.

c) Write the standard basis for vector space $\mathbb{R}^n$.

d) Find the characteristic equation of matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$.

e) Determine x and y if $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}, B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$ and $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$.

f) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation for which $T(1, 1) = (1, -2)$ and $T(-1, 1) = (2, 3)$ $then\ find$ $T(-1, 5)$.

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