BACHELOR OF SCIENCE (COMPUTER SCIENCE) (CBCS - 2018 COURSE)
F.Y.B.Sc.(Computer Science) Sem-II :SUMMER- 2022
SUBJECT : ALGEBRA-II

Day : Friday
Date : 8/7/2022

S-20081-2022

Time : 11:00 AM-02:00 PM
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.

Q.1 Attempt any **TWO** of the following: **(12)**

a) Let $\mathbb{Q}^+$ be the set of all positive rational numbers, define binary operation $*$ on $\mathbb{Q}^+$ as $a*b=\frac{ab}{2}, \forall a, b \in \mathbb{Q}^+$. Show that $(\mathbb{Q}^+,*)$ is a group.

b) Show that $S=\{1,-1, i,-i\}$ forms a group under multiplication.

c) Find all subgroups of cyclic group of order 12. Draw Hasse diagram for subgroup relation.

Q.2 Attempt any **TWO** of the following: **(12)**

a) State and prove Lagrange's Theorem.

b) Construct composition table for group S_3.

c) Let $\sigma_1=\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 2 & 4 & 1\end{pmatrix}$ and $\sigma_2=\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix}$, find σ_1^{-1}, σ_2^{-1} and also compute $\sigma_1 o\,\sigma_2$ and $\sigma_2 o\,\sigma_1$. *Is* $\sigma_1 o\,\sigma_2=\sigma_2 o\,\sigma_1$?

Q.3 Attempt any **TWO** of the following: **(12)**

a) Prove that every infinite cyclic group is isomorphic to $(\mathbb{Z},+)$.

b) Find all generators of cyclic group $(\mathbb{Z}_{30},+)$.

c) Express the permutation $\sigma=\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ 2 & 5 & 4 & 3 & 6 & 1 & 7 & 9 & 8\end{pmatrix}$ as a product of disjoint cycles. Determine σ is even or odd.

Q.4 Attempt any **THREE** of the following: **(12)**

a) Construct addition table for $(\mathbb{Z}_8,+_8)$.

b) If G is a group then prove that every element has unique inverse in G.

c) Is union of two subgroups is subgroup? Justify.

d) Let $f:(\mathbb{Z}, +)\to(\mathbb{Z}_n, +_n)$ be a function defined by $f(a)=\bar{a}$. Show that f is homomorphism. Find $\ker(f)$.

Q.5 Attempt any **FOUR** of the following: **(12)**

a) Define the term 'order of an element' and find $o(\bar{2})$ in the group $(\mathbb{Z}_6,+_6)$.

b) Find all subgroups of S_3.

c) Show that the group G is abelian if and only if $(a\,b)^2=a^2\,b^2, \forall a, b\in G$.

d) Prove that every proper subgroup of a group of order 51 is cyclic.

e) Let $\phi:(\mathbb{Z}, +)\to(\mathbb{Z}, +)$ define as $\phi(n)=2n, \forall n\in\mathbb{Z}$ verify ϕ is group homomorphism.

f) Show that A_3 is normal subgroup is S_3 .

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