BACHELOR OF SCIENCE (COMPUTER SCIENCE) (CBCS - 2018 COURSE)
F.Y.B.Sc.(Computer Science) Sem-I :SUMMER- 2022
SUBJECT : ALGEBRA-I

Day : Saturday
Date : 9/7/2022

S-20069-2022

Time : 11:00 AM-02:00 PM
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.

Q.1 Attempt **ANY TWO** of the following: **[12]**

a) State De Moivre's theorem and use it to prove $\left(1+i\sqrt{3}\right)^{-10} = 2^{-11}\left(-1+i\sqrt{3}\right)$.

b) Express $\cos^7\theta$ and $\sin^7\theta$ in terms of the cosines of multiple angles.

c) If $z_1, z_2 \in \mathbb{C}$ then prove that,

i) $|z_1 \ z_2| = |z_1|\,|z_2|$

ii) $\arg(z_1 \ z_2) = \arg z_1 + \arg z_2$

Q.2 Attempt **ANY TWO** of the following: **[12]**

a) Let S = {1, 2, 3, 4, 5} and R = {(1, 2), (3, 4), (3, 2), (4, 5), (5, 3), (1, 5)} be a relation on S. Find the transitive closure of R by using Warshall's algorithm.

b) If the function $f:\mathbb{R}\to\mathbb{R}$ define as $f(x) = \dfrac{2x-3}{7}$ $\forall x \in \mathbb{R}$, then show that f is bijective. Hence find f^{-1}.

c) If a, b, $x \in \mathbb{Z}$, $n \in \mathbb{N}$ and a ≡ b (mod n) then prove that,

i) (a + x) ≡ (b + x) (mod n).

ii) ax ≡ bx (mod n).

Q.3 Attempt **ANY TWO** of the following: **[12]**

a) Show that a = 389 and b = 167 are relatively prime. Also find integers x and y such that 389x + 167y = 1.

b) If p is a prime integer and a, b $\in \mathbb{Z}$ then prove that if p | ab then p | a or p | b.

c) Construct a decoding table with syndromes for a group code given by generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$.

Use the table to decode the following received word:

i) 11110 **ii)** 10010.

P.T.O.

Q.4 Attempt **ANY THREE** of the following: **[12]**

a) Obtain the remainder when 8^{401} is divided by 13.

b) Prove that for any integer x, (a, b) = (a, b + ax).

c) Solve $x^8 - x^4 + 1 = 0$ by De Moivre's theorem.

d) Construct decoding table for the (2, 4) codes given by the following generator matrix, $G = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

Q.5 Attempt **ANY FOUR** of the following: **[12]**

a) Give an example of a relation which is:
i) symmetric but neither reflexive nor transitive.
ii) equivalence

b) If $f:\mathbb{R}\to\mathbb{R}$ is define by $f(x)=x^2+2x+3$ and $g:\mathbb{R}\to\mathbb{R}$ is define by $g(x)=2x+3$, find fog=?

c) Express the following complex number into polar form, $z=\frac{-1-i\sqrt{3}}{2}$.

d) If $z+\frac{1}{z}$ is real then show that I(z) = 0 or |z| = 1.

e) Prove that $\sqrt{5}$ is not a rational number.

f) Find the Hamming distance between x = 00000 and y = 11111.

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