BACHELOR OF SCIENCE (CBCS-2018 COURSE)
S. Y. B. Sc. Sem-IV :SUMMER- 2022
SUBJECT : MATHEMATICS : COMPLEX VARIABLES

Day : Wednesday — Time : 03:00 PM-06:00 PM

Date : 13-07-2022 — **S-18393-2022** — Max. Marks : 60

N.B.

1) All questions are **COMPULSORY**.
2) Figures to the **RIGHT** indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.

Q.1 Attempt **ANY TWO** of the following : **(12)**

a) Show that a necessary condition for a function w = f(z) = u(x, y) + iv (x, y) be analytic at a point z = x+iy is that at (x, y) the real and imaginary parts u and v of f(z) satisfy the Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$.

b) Prove that if $\lim_{z\to z_0} f(z)$ exists, then it is unique.

c) Find an analytic function whose real part is $2x - x^3 + 3xy^2$.

Q.2 Attempt **ANY TWO** of the following : **(12)**

a) Evaluate : $\int_c \frac{z+6}{z^2-4}\, dz$, where

i) c is the circle $|z| = 1$

ii) c is the circle $|z-2| = 1$

iii) c is the circle $|z+2| = 1$.

b) Expand the following functions in a Taylor's series about z = 0

i) f (z) = sin z

ii) g (z) = cos z .

c) Obtain the expansion of $f(z)=\frac{1}{z^2-3z+2}$ in the region $1<|z|<2$.

Q.3 Attempt **ANY TWO** of the following : **(12)**

a) Prove that if $f(z)$ has a simple pole at $z=z_0$ then the residue of $f(z)$ at $z=z_0$ is $\lim_{z\to z_0} (z-z_0)\, f(z)$.

b) Evaluate $\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)}\, dx$ by contour integration .

c) If both $f(z)$ and $\overline{f(z)}$ are analytic functions of z then prove that $f(z)$ is constant.

Q.4 Attempt **ANY THREE** of the following : **(12)**

a) Prove that the sum of the residues of the function $\frac{e^z}{z^2+a^2}$ is $\frac{\sin a}{a}$.

b) Obtain the Laurent's series of the function

$f(z)=\frac{z^2-1}{(z+2)(z+3)}$ in the region $2<|z|<3$

c) Evaluate $\lim_{z\to e^{\frac{\pi i}{4}}} \frac{z^2}{z^4+z^2+1}$.

d) Evaluate $\int_C (x^2+y^2-xy\, i)\, dz$ where C is the line segment from $z=0$ to $z=1+i$.

P.T.O.

Q.5 Attempt **ANY FOUR** of the following : **(12)**

a) Evaluate $\lim_{z \to i} \frac{iz^3 - 1}{z + i}$.

b) Show that a function $f(z) = \bar{z}$ is continuous everywhere but differentiable nowhere .

c) Evaluate $\int_c \frac{z+6}{z^2 - 2} dz$, where c is the circle $|z| = 1$.

d) Find the zeros of $\left(z^4 + 8z^2 + 16\right)\left(z^2 + z + 1\right)$.

e) Discuss the continuity of the function $f(z)$ at $z = 2i$, if

$$f(z) = \frac{z^2 + 4}{z - 2i}, \quad if \ z \neq 2i$$
$$= 3 + 4i, \quad if \ z = 2i.$$

f) Define pole of the function with an example.

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