BACHELOR OF SCIENCE (CBCS-2018 COURSE)
S. Y. B. Sc. Sem-IV :SUMMER- 2022
SUBJECT : MATHEMATICS : VECTOR CALCULUS

Day : Monday
Date : 11/7/2022

S-18392-2022

Time : 03:00 PM-06:00 PM
Max. Marks : 60

N.B.
1) All questions are **COMPULSORY**.
2) Figures to the **RIGHT** indicate **FULL** marks.
3) Use of non-programmable calculator is **ALLOWED**.

Q.1 Attempt **ANY TWO** of the following : **(12)**

a) A non-constant vector function $\bar{u}(t)$ is of constant direction if and only if $\bar{u} \times \frac{d\bar{u}}{dt} = \bar{0}$.

b) If $\bar{r} = \bar{a}\cos\omega t + \bar{b}\sin\omega t$, where $\bar{a}$ and $\bar{b}$ are constant vectors and ω is a constant scalar, prove that: i) $\bar{r} \times \frac{d\bar{r}}{dt} = \omega\left(\bar{a} \times \bar{b}\right)$ and ii) $\frac{d^2\bar{r}}{dt^2} = -\omega^2\bar{r}$.

c) If $\bar{f} = (uvw)\hat{i} + \left(uw^2\right)\hat{j} - v^3\hat{k}$ and $\bar{g} = u^3\hat{i} - (uvw)\hat{j} + (u^2w)\hat{k}$ then find $\frac{\partial^2\bar{f}}{\partial v^2} \times \frac{\partial^2\bar{g}}{\partial u^2}$ at $(1, 1, -1)$.

Q.2 Attempt **ANY TWO** of the following : **(12)**

a) Find the acute angle between the tangents to the curve $\bar{r} = t^2\hat{i} + 2t\hat{j} - \frac{t^2}{2}\hat{k}$ at the points t = 1 and t = –3 .

b) If $\bar{r} = a\cos u\sin t\,\hat{i} + a\sin u\cos t\,\hat{j} + a\cos t\,\hat{k}$, show that $\frac{1}{a}\left(\frac{\partial\bar{r}}{\partial u} \times \frac{\partial\bar{r}}{\partial t}\right)$ is a unit vector.

c) Find the scalar function $\phi(x, y, z)$ if
$\nabla\phi = \left(y^2 - 2xyz^3\right)\hat{i} + \left(3 + 2xy - x^2z^3\right)\hat{j} + \left(6z^3 - 3x^2yz^2\right)\hat{k}$ and $\phi(1, 1, 2) = 0$.

Q.3 Attempt **ANY TWO** of the following : **(12)**

a) State and prove Green's theorem in the plane.

b) Evaluate $\int_C \left[\left(x^2 - y^2\right)\hat{i} + 2xy\hat{j}\right].d\bar{r}$ around a rectangle with vertices at (0, 0), (a, 0), (a, b) and (0, b) traversed in counter-clockwise direction.

c) Verify the divergence theorem for $\bar{f} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4$, z = 0 and z = 3 .

P.T.O.

Q.4 Attempt **ANY THREE** of the following : **(12)**

a) Let $\bar{u}$ be the vector point function then show that div (curl $\bar{u}$) = 0 .

b) Find the directional derivative of the function $\phi(x,y,z) = x^2 y^3 - 2xz^2 + 3$ at the point P (2, 1,–2) in the direction towards Q (4, 0, 3) .

c) If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\bar{r}| = \sqrt{x^2 + y^2 + z^2}$, find : i) ∇r ii) div $\bar{r}$.

d) Find the equations of the tangent plane and the normal to the surface $xy + yz + xz = 7$ at (1, 1, 3) .

Q.5 Attempt **ANY FOUR** of the following : **(12)**

a) If $\bar{r} = e^{-t}\hat{i} + \log(t^2+1)\hat{j} - \tan t\,\hat{k}$ find:

i) $\dfrac{d\bar{r}}{dt}$ ii) $\dfrac{d^2\bar{r}}{dt^2}$ iii) $\left|\dfrac{d\bar{r}}{dt}\right|$ at t = 4 .

b) Find the differential equation whose solution is $\bar{r} = \bar{a}e^{5t} + \bar{b}e^{-t}$ where $\bar{a}$ and $\bar{b}$ are constant vectors.

c) If $\bar{r} = \frac{a}{2}(x+y)\hat{i} + \frac{b}{2}(x-y)\hat{j} + xy\hat{k}$ find : $\left[\dfrac{\partial \bar{r}}{\partial x}, \dfrac{\partial \bar{r}}{\partial y}, \dfrac{\partial^2 \bar{r}}{\partial x^2}\right]$.

d) Show that $\bar{u} = x^2 z\hat{i} + y^2 z\hat{j} - (xz^2 + yz^2)\hat{k}$ is solenoidal.

e) Define gradient of a scalar point function and divergence of a vector point function.

f) Evaluate the line integral of $\bar{f} = x^2\hat{i} - xy\hat{j}$ from O (0,0) to P (1,1) along the straight path OP.

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