BACHELOR OF SCIENCE (COMPUTER SCIENCE) (CBCS - 2018 COURSE)
S.Y.B.Sc.(Computer Science) Sem-III : WINTER :- 2021
SUBJECT: LINEAR ALGEBRA

Day : Saturday
Date 22-01-2022

W-20093-2021

Time : 10:00 AM-01:00 PM
Max. Marks: 60

N.B.

1) All questions are **COMPULSORY**.
2) Figures to the **RIGHT** indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following. **(12)**

a) Let $S=\{\overline{v_1},\overline{v_2},..........,\overline{v_n}\}$ be a set of vectors in a vector space. Prove that S is linearly dependent if and only if one of the vectors in S is a linear combination of the remaining vectors in S.

b) Find the coordinate vector of $\overline{u}=(-1,4,1)$ relative to the basis, B ={(1, 1, 0), (1, 0, 1), (0, 1, 1)} .

c) Find a basis and dimension for the column space of matrix A, where $A=\begin{bmatrix} 2 & 3 & 5 & 7 & 4 \\ -1 & 2 & 1 & 0 & -2 \\ 4 & 1 & 5 & 9 & 8 \end{bmatrix}$.

Q.2 Attempt **ANY TWO** of the following. **(12)**

a) Prove that if λ is an eigenvalue of a square matrix A, then λ^m is an eigenvalue of A^m for every positive integer m.

b) Find all the eigenvalues of matrix A and also find eigenspace corresponding to the largest eigenvalue, where $A=\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$.

c) Find matrix P that diagonalize the matrix $A=\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$.

Q.3 Attempt **ANY TWO** of the following. **(12)**

a) Let $T: V \to W$ is a linear transformation then prove that
   i) kernel of T is a subspace of V
   ii) range of T is a subspace of W.

b) Find standard matrix for the linear transformation $T:\mathbb{R}^4 \to \mathbb{R}^3$ is defined by, $T(x_1, x_2, x_3, x_4) = (3x_1-x_2+x_3, x_2+x_4, 4x_1+x_3)$.

c) If $T:\mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that T(1, 0, 0) = (0, 0) , T(0, 1, 0) = (1, 1), T(0, 0, 1) = (1,–1) then find :
i) rank (T)   ii) nullity (T)   iii) T (4,–1, 1) = ?

Q.4 Attempt **ANY THREE** of the following. **(12)**

a) If $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is any 2×2 matrix. Show that if A is invertible, then $(ad-bc)\neq 0$ and $A^{-1}=\frac{1}{(ad-bc)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

b) Find LU factorization of the coefficient matrix of the given linear system $A\overline{X}=\overline{b}$. Solve the linear system using a forward substitution followed by back substitution, where $A=\begin{bmatrix} 2 & 3 & 4 \\ 4 & 5 & 10 \\ 4 & 8 & 2 \end{bmatrix}$,$\overline{b}=\begin{bmatrix} 6 \\ 16 \\ 2 \end{bmatrix}$.

c) For what value of a does the following system has :

i) Unique solution

ii) Infinitely many solution.

$(a-3)x+y=0$

$x+(a-3)y=0$

d) Solve the following system of equations :

$2x_1+3x_2+x_3=1$

$x_1+x_2+x_3=3$

$3x_1+4x_2+2x_3=4$

Q.5 Attempt **ANY FOUR** of the following. **(12)**

a) If V is a vector space then show that

i) $0\bar{u}=\bar{0},\ \forall \bar{u}\in V$.

ii) $(-1)\bar{u}=-\bar{u},\ \forall \bar{u}\in V$.

b) Define: i) Vector space ii) Subspace

c) Find the dot product of $\bar{a}=[1, 2, 3]$ and $\bar{b}=[2,4,9]$.

d) Let $A=\begin{bmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{bmatrix}$. Find the eigenvalues of A^T.

e) Determine whether $T:\mathbb{R}^2\to\mathbb{R}^3$ defined by $T(x,y)=(x+y,\ x-y,1)$ is a linear transformation or not?

f) Reduce the matrix $A=\begin{bmatrix} 2 & 1 & 5 & 2 \\ 0 & -1 & 0 & 2 \\ 1 & 2 & 5 & 6 \end{bmatrix}$ into row echelon form.

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