BACHELOR OF SCIENCE (COMPUTER SCIENCE) (CBCS - 2018 COURSE)
F.Y.B.Sc.(Computer Science) Sem-II : WINTER :- 2021
SUBJECT: ALGEBRA-II

Day : Monday
Date 24-01-2022

W-20081-2021

Time : 02:00 PM-05:00 PM
Max. Marks: 60

N.B.
1) All questions are **COMPULSORY**.
2) Figures to the **RIGHT** indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **(12)**

a) Let G be a group, if a, $b \in G$ then prove that $(a \cdot b)^{-1} = b^{-1}a^{-1}$.

b) Let $Q_1 = Q - \{1\}$. Define composition $*$ on Q_1 as $a * b = a + b - ab \ \forall\ a,b \in Q_1$. Prove that $(Q_1, *)$ is an abelian group.

c) Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 5 & 4 & 3 & 6 & 1 & 7 & 9 & 8 \end{pmatrix}$ as a product of disjoint cycles. Determine whether σ is even or odd.

Q.2 Attempt **ANY TWO** of the following: **(12)**

a) State and prove Lagrange's theorem.

b) Show that $H = \left\{ \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix} / a \neq 0 \text{ where } a, b \in \mathbb{R} \right\}$ be a subgroup of $M_2(\mathbb{R})$ the multiplicative group of 2×2 non-singular matrices over $\mathbb{R}$.

c) Find all the subgroups of group $\mathbb{Z}_{36}$ and draw a Hasse diagram for these subgroup relations.

Q.3 Attempt **ANY TWO** of the following: **(12)**

a) A subgroup H of a group G is normal if and only if $xHx^{-1} = H \ \forall x \in G$.

b) Let G be the six element group {e, a, b, c, d, f }

	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	b	e	d	f	c
b	b	e	a	f	c	d
c	c	f	d	e	b	a
d	d	c	f	a	e	b
f	f	d	c	b	a	e

Find normal subgroup of G.

c) Show that the multiplicative group $G = \{1, \omega, \omega^2\}$ where ω is complex cube root of unity is isomorphic to $(\mathbb{Z}_3, +_3)$.

Q.4 Attempt **ANY THREE** of the following: **(12)**

a) Show that A_3 is normal in S_3.

b) State and prove left cancellation law and right cancellation law in a group.

c) Find the order of every element in $(Z_4, +_4)$.

d) Write down factor group $\frac{\mathbb{Z}_6}{<2>}$.

P.T.O.

Q.5 Attempt **ANY FOUR** of the following: **(12)**

a) If a is any element in a group G then show that $(a^{-1})^{-1} = a$.

b) Prove that every cyclic group is abelian.

c) Find all the generators of group $(\mathbb{Z}_5, +_5)$.

d) Write down the following permutation as a product of disjoint cycles.
$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 5 & 7 & 6 & 2 & 4 & 1 & 3 \end{pmatrix}$.

e) Define: **i)** Ring **ii)** Field **iii)** Integral Domain.

f) Show that $(\mathbb{Z}, +)$ is isomorphic to $(m\mathbb{Z}, +)$.

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