

BACHELOR OF SCIENCE (COMPUTER SCIENCE) (CBCS - 2018 COURSE)
F.Y.B.Sc.(Computer Science) Sem-I : WINTER :- 2021
SUBJECT: ALGEBRA-I

Day : Monday
Date 24-01-2022

W-20069-2021

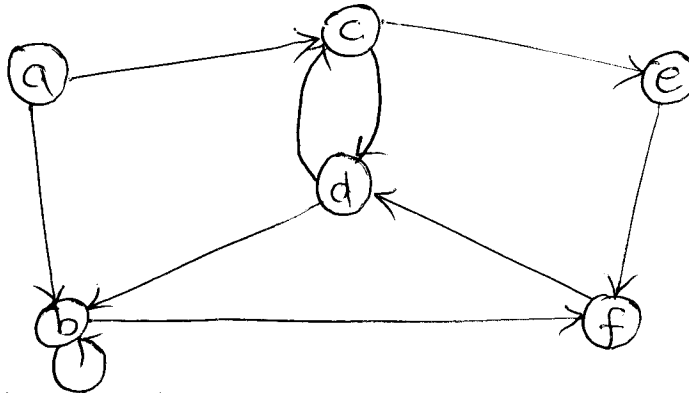
Time : 10:00 AM-01:00 PM
Max. Marks: 60

N.B.

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following : **(12)**

- a) Find the transitive closure of the relation given below:



- b) Prove that R is equivalence relation, if R be a relation on \mathbb{Z} defined by aRb if and only if $a - b$ is divisible by 5, for $a, b \in \mathbb{Z}$.
- c) Prove that if $z_1, z_2 \in \mathbb{C}$ then
- i) $|z_1 z_2| = |z_1| |z_2|$
 - ii) $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$.

Q.2 Attempt **ANY TWO** of the following : **(12)**

- a) State De-Moivre's theorem and prove it for positive and negative case.
- b) Prove that $(1 + i\sqrt{3})^{-10} = 2^{-11} (-1 + i\sqrt{3})$.
- c) If $a, b, x \in \mathbb{Z}$, $n \in \mathbb{N}$ and $a \equiv b \pmod{n}$ then prove that
- i) $(a + x) \equiv (b + x) \pmod{n}$
 - ii) $ax \equiv bx \pmod{n}$.

Q.3 Attempt **ANY TWO** of the following : **(12)**

- a) Find the g.c.d. of 1357 and 1166 and express the g.c.d in the form $1357m + 1166n$.
- b) Obtain the remainder when 8^{401} is divided by 13.

P.T.O.

c) Let $H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix for hamming (7, 4)

code. Decode the following received words i) 1100001 ii) 1110111.

Q.4 Attempt **ANY THREE** of the following : **(12)**

- a) If $c \mid ab$ and $(b, c) = 1$ then prove that $c \mid a$.
- b) If p is a prime number and a, b are integers, prove that if $p \mid ab$ then $p \mid a$ or $p \mid b$.
- c) If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a function defined by $f(x) = 3x - 2$. Show that f is bijective function and hence find formula for f^{-1} .
- d) Construct a decoding table for a (2,4) codes given by the following generator

matrix $G = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

Q.5 Attempt **ANY FOUR** of the following : **(12)**

- a) Find real and imaginary part of $z = \frac{1}{1-i}$.
- b) Let $a = 175, b = 300$. Find (a, b) and $[a, b]$.
- c) Draw a digraph of RoR where, $R = \{ (1, 2), (2, 1), (3, 4), (4, 3), (3, 5), (5, 3), (4, 5), (5, 4), (5, 5) \}$ on $A = \{ 1, 2, 3, 4, 5 \}$
- d) Define relatively prime integers with example.
- e) Determine whether the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2$ is onto or not?
- f) Find the minimum distance d for the following code:

$C = \{ 1101, 1001, 0110, 1110, \}$ in \mathbb{Z}_2^4 .

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