BACHELOR OF SCIENCE (CBCS-2018 COURSE)
S. Y. B. Sc. Sem-IV : WINTER :- 2021
SUBJECT: MATHEMATICS : COMPLEX VARIABLES

Day : Monday
Date 31-01-2022

W-18393-2021

Time : 02:00 PM-05:00 PM
Max. Marks: 60

N.B.

1) All questions are **COMPULSORY**.
2) Figures to the **RIGHT** indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.

Q.1 Attempt **ANY TWO** of the following : **(12)**

a) Let f and g be the functions whose limit exists at z_0 and if $\lim_{z\to z_0} f(z)=\omega_0$ and $\lim_{z\to z_0} g(z)=W_0$ then prove that i) $\lim_{z\to z_0}[f(z)+g(z)] = \omega_0 + W_0$

ii) $\lim_{z\to z_0} f(z)\cdot g(z) = \omega_0\, W_0$.

b) If $f(z)=\dfrac{3z^4-2z^3+8z^2-2z+5}{z-i}$, if $z_0=i$ and $f(i)=2+3i$, examine $f(z)$ for continuity at $z=i$.

c) Prove that the function defined by $f(z)=\bar{z}$ is continuous everywhere but differentiable nowhere.

Q.2 Attempt **ANY TWO** of the following : **(12)**

a) Using Cauchy Gourast theorem, obtain the value of $\int_C e^z dz$, where C is the circle |z| = 1 and show that i) $\int_0^{2\pi} e^{\cos\theta}\left[\sin(\theta+\sin\theta)\right] d\theta=0$

ii) $\int_0^{2\pi} e^{\cos\theta}\left[\cos(\theta+\sin\theta)\right] d\theta=0$.

b) Evaluate $\int_C \dfrac{z^3}{z-2i} dz$, where C is the circle $|z-2|=5$ by using Cauchy integral formula.

c) Expand the following functions in a Taylor's series about z = 0

i) f (z) = sin z
ii) g (z) = cos z .

Q.3 Attempt **ANY TWO** of the following : **(12)**

a) Evaluate by contour integration $\int_C \dfrac{5z-2}{z(z-1)} dz$, where C is the circle |z|=3 taken counter-clockwise .

b) Prove that $\dfrac{1}{2\pi i}\int_C \dfrac{e^{\lambda z}}{z^2+1} dz = \sin\lambda$, where $\lambda>0$ and C is the circle |z|=3 .

c) Evaluate $\int_{-\infty}^{\infty} \dfrac{dx}{x^2+1}$ by applying calculus of residues .

P.T.O.

Q.4 Attempt **ANY THREE** of the following : **(12)**

a) Prove that $\lim_{z\to 0}\frac{\bar{z}}{z}$ does not exists.

b) If a function f of a complex variable z is differentiable at z_0 then it is continuous at z_0 .

c) Evaluate $\int_C \left(x^2+iy^3\right)dz$, where C is the line segment from z = 1 to z = i .

d) Evaluate $\int_C \frac{e^z}{(z+1)^2}\,dz$, where C is the circle $|z-1| = 3$.

Q.5 Attempt **ANY FOUR** of the following : **(12)**

a) Evaluate $\lim_{z\to(1+i)} \frac{z^2-z+1-i}{z^2-2z+2}$.

b) State Cauchy Gourast theorem.

c) Show that the function $u=\frac{1}{2}\log(x^2+y^2)$ is Harmonic.

d) Determine poles and their orders for the function $f(z)=\frac{z+3}{z(z^4-1)}$.

e) Find the residue of $f(z)=\frac{1}{(z^2+1)^3}$ at z = i .

f) State Cauchy integral formula for $f(z_0)$.

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