

**BACHELOR OF SCIENCE (CBCS-2018 COURSE)**  
**S. Y. B. Sc. Sem-III : WINTER :- 2021**  
**SUBJECT: MATHEMATICS : GROUP THEORY & DIFFERENTIAL**  
**EQUATIONS**

Day : Wednesday  
Date 2/2/2022

W-18363-2021

Time : 10:00 AM-01:00 PM  
Max. Marks: 60

**N.B.**

- 1) All questions are **COMPULSORY**.
- 2) Figures to the **RIGHT** indicate **FULL** marks.

**Q.1** Attempt **ANY TWO** of the following. **(12)**

- a) Show that if  $a, b$  are any two elements in a group  $G$  then  $O(a) = O(b^{-1}ab)$ .
- b) Show that the set  $S = \{1, \omega, \omega^2\}$  where  $\omega$  is a complex cube root of unity is a group under multiplication of complex numbers.
- c) Show that the group  $G$  is abelian if and only if  $(ab)^2 = a^2b^2, \forall a, b \in G$ .

**Q.2** Attempt **ANY TWO** of the following. **(12)**

- a) Prove that a non-empty subset  $H$  of a group  $G$  is a subgroup of  $G$  if and only if  $ab^{-1} \in H, \forall a, b \in H$ .
- b) Let ' $a$ ' be any element of a group  $G$ . Define  $N(a) = \{x \in G \mid xa = ax\}$ . Prove that  $N(a)$  is a subgroup of  $G$ .
- c) If  $H$  is a subgroup of a finite group  $G$ , then prove that  $O(H) \mid O(G)$ .

**Q.3** Attempt **ANY TWO** of the following. **(12)**

- a) Show that  $\frac{1}{f(D^2)} \sin(ax+b) = \frac{\sin(ax+b)}{f(-a^2)}$  where  $f(-a^2) \neq 0$ .
- b) Solve :  $(D^3 + 4D)y = \cos 2x$ .
- c) Solve :  $(D^3 - 2D + 4)y = x^4 + 3x^2 - 5x + 2$ .

**Q.4** Attempt **ANY THREE** of the following. **(12)**

- a) Show that by using the substitutions  $x^2 = u$  and  $y^2 = v$ , a differential equation  $(px - y)(py + x) = 2p$  becomes Clairaut's equation and hence evaluate it.
- b) Solve :  $x^2 p^2 + xyp - 6y^2 = 0$ .
- c) Solve :  $4y = x^2 + p^2$ .
- d) Solve :  $e^{3x}(p-1) + p^3 e^{2y} = 0$ .

**Q.5** Attempt **ANY FOUR** of the following. **(12)**

- a) Prove that in any group there is a unique identity element.
- b) Show that  $(Z_5, +_5)$  is a cyclic group. Find all its generators.
- c) Find all the subgroups of a cyclic group of order 32.
- d) Find particular integral of  $(D^3 + 3D^2 + 3D + 1)y = e^{-x}$ .
- e) Solve :  $p^2 - 7p + 12 = 0$ , where  $p = \frac{dy}{dx}$ .
- f) Solve :
  - i)  $y - 2px = f(p^2)$ .
  - ii)  $(y - px)^2 = 1 + p^2$

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