BACHELOR OF SCIENCE (CBCS-2018 COURSE)
S. Y. B. Sc. Sem-III : WINTER :- 2021

SUBJECT: MATHEMATICS : CALCULUS OF SEVERAL VARIABLES

Day : Monday
Date 31-01-2022

W-18362-2021

Time : 10:00 AM-01:00 PM
Max. Marks: 60

N.B.

1) All questions are **COMPULSORY**.
2) Figures to the **RIGHT** indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following. **(12)**

a) Show that if f be a real-valued function defined on a neighbourhood of (a,b) and f is differentiable at (a,b) then,

i) f is continuous at (a,b)

ii) f_x (a,b) and f_y (a,b) both exist.

b) If $f = \tan^{-1}\left(\frac{y}{x}\right)$ then verify $\frac{\partial^2 f}{\partial y\,\partial x} = \frac{\partial^2 f}{\partial x\,\partial y}$

c) If $u = f(x,y)$ and $x = r\cos\theta$, $y = r\sin\theta$ then show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2 .$$

Q.2 Attempt **ANY TWO** of the following. **(12)**

a) State and prove Taylor's theorem for a function of two variables x and y.

b) Expand $f(x,y) = \sin xy$ in powers of $(x-1)$ and $\left(y - \frac{\pi}{2}\right)$.

c) If $f(x,y) = \frac{x^3 y}{x^2 + y^2}$, $x^2 + y^2 \neq 0$

and $f(0,0) = 0$ then show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

Q.3 Attempt **ANY TWO** of the following. **(12)**

a) Explain Lagrange's method of undetermined multipliers.

b) Divide the number 24 into three equal parts so that the continued product of the first, square of the second and the cube of the third may be maximum.

c) Find the extreme values of the function $f(x,y) = xy(a-x-y)$

Q.4 Attempt **ANY THREE** of the following. **(12)**

a) Find the volume of the ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$ by triple integration.

b) Evaluate $\iint_D \sqrt{1-x^2-y^2}\,dx\,dy$ where D is bounded by $x = 0$, $y = 0$ and $x^2 + y^2 = 1$ in the first quadrant.

c) Evaluate : $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dx\,dy\,dz}{(1+x+y+z)^3}$.

d) Change the order of integration and hence evaluate; $\int_0^x \int_0^x \frac{e^{-y}}{y}\,dy\,dx$.

Q.5 Attempt **ANY FOUR** of the following. **(12)**

a) Show that the function $f(x,y)$ defined by $f(x,y) = \frac{x^2 y^2}{x^2 + y^2}$, $(x,y) \neq (0,0)$

$f(0,0) = 0$

is continuous at (0, 0).

b) Change the order of integration in $\int_0^a \left[\int_0^{\sqrt{a^2-x^2}} f\,dy\right] dx$.

c) If $f(x,y) = 2x^4 - 5xy^3$ then find f_{xx} and f_{yy} at the point (1, 1).

d) Prove that $\int_0^1 \int_0^1 (x^2 + y^2)\,dx\,dy = \frac{2}{3}$.

e) Evaluate: $\lim_{(x,y)\to(0,0)} \frac{x^2 y^2}{x^2 y^2 + (x-y)^2}$, along the path $y = x$

f) Define: i) Maximum value
ii) Minimum value
iii) Extreme value

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