BACHELOR OF SCIENCE (CBCS-2018 COURSE)
S. Y. B. Sc. Sem-III : WINTER :- 2021
SUBJECT: PHYSICS : MATHEMATICAL METHODS FOR PHYSICS

Day : Thursday
Date 20-01-2022

W-18347-2021

Time : 10:00 AM-01:00 PM
Max. Marks: 60

N.B.

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **(12)**

a) If $z=\dfrac{1+\sqrt{3i}}{2}$, evaluate $z^3, |z|$.

b) Find the first and second order partial derivatives of the function $F=f(x,y)=2x^3y^2+y^3$.

c) Show that $\vec{\nabla}\times\left(\vec{A}+\vec{B}\right)=\vec{\nabla}\times\left(\vec{A}\right)+\vec{\nabla}\times(\vec{B})$.

Q.2 Attempt **ANY TWO** of the following: **(12)**

a) Determine the directional derivative of $\phi=4xz-3xy^2+zy^2x$ at $(1,-1,2)$ in the direction of $\left(\hat{i}-2\hat{j}+\hat{k}\right)$.

b) Determine a unit vector perpendicular to the plane of $\vec{A}=2\hat{i}+6\hat{j}-3\hat{k}$ and $\vec{B}=4\hat{i}+3\hat{j}-\hat{k}$

c) Determine different values of the fifth root of $1+i\sqrt{3}$.

Q.3 Attempt **ANY TWO** of the following: **(12)**

a) If $F=a\ln(x^2+y^2)$, show that $F_{xy}=F_{yx}$ and $F_{xx}+F_{yy}=0$.

b) Show that the curl of the linear velocity of any particle of rotating body is twice its angular velocity.

c) If $\vec{A}=2\hat{i}+3\hat{j}+5\hat{k}$, $\vec{B}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{C}=2\hat{i}+\hat{j}+3\hat{k}$, show that $\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=\vec{B}\cdot\left(\vec{C}\times\vec{A}\right)=\vec{C}.\left(\vec{A}\times\vec{B}\right)$

Q.4 Attempt **ANY THREE** of the following: **(12)**

a) Show that $\vec{F}=\cos y\,\hat{i}-x\sin y\,\hat{j}-\cos z\,\hat{k}$ is a conservative field.

b) Show that the vectors $\vec{A}=3\hat{i}-2\hat{j}+\hat{k}$, $\vec{B}=\hat{i}-3\hat{j}+5\hat{k}$ and $\vec{C}=2\hat{i}+\hat{j}-4\hat{k}$, form a right angled triangle.

c) Find the percentage error in the area of ellipse when an error of 1% is made in measuring its major and minor axes.

d) Calculate the value of the product $\left(\cos\dfrac{3\pi}{7}+i\sin\dfrac{3\pi}{7}\right)\left(\cos\dfrac{2\pi}{7}+i\sin\dfrac{2\pi}{7}\right)^2$.

Q.5 Attempt **ANY FOUR** of the following: **(12)**

a) Define linearity and homogeneity of differential equation.

b) Determine the volume of parallelepiped defined by vectors $\vec{A}=2\hat{i}-\hat{j}-\hat{k}$, $\vec{B}=\hat{i}-2\hat{j}-3\hat{k}$ and $\vec{C}=3\hat{i}+2\hat{j}+5\hat{k}$.

c) Find order and degree of differential equation $\dfrac{d^3y}{dx^3}+x\left(\dfrac{dy}{dx}\right)^{3/2}+x^2y=0$.

d) Prove that grad $(\phi+\psi)=\text{grad}\,\phi+\text{grad}\,\psi$, where ϕ and ψ are differentiable scalar function of x, y and z.

e) Find the equation of the tangent line to $x^3-3y^3+xy+2=0$ at points $(1, 2)$.

f) Transform $\dfrac{1}{(1-i)^2}$ to exponential form.

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