BACHELOR OF SCIENCE (CBCS-2018 COURSE)
F. Y. B. Sc. Sem-II : WINTER :- 2021

SUBJECT: STATISTICS : DISCRETE PROBABILITY & PROBABILITY DISTRIBUTIONS-II

Day : Saturday
Date 5/2/2022

W-18338-2021

Time : 02:00 PM-05:00 PM
Max. Marks: 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of statistical tables and **CALCULATOR** is allowed.

Q.1 Attempt **ANY TWO** of the following: **[12]**

a) State and prove additive property of two independent Poisson random variables (r.v.s).

b) If X follows Poisson (m) such that $P(X = 2) = \frac{1}{2} P(X = 1)$, find probability mass function (p.m.f.) of X. Also find mean and variance of X, $P(X \geq 3)$ and $P(3 < X < 5)$.

c) Let X be a discrete r.v. with p.m.f.

$$p(x) = pq^{x} \quad ; \quad x = 0,1,2,.... ; \quad 0 < p < 1,\ q = 1 - p$$
$$= 0 \quad ; \quad \text{otherwise}$$

Find E(X) and Var (X).

Q.2 Attempt **ANY TWO** of the following: **[12]**

a) Define the joint distribution function of a two dimensional discrete r.v. Also state its important properties.

b) The joint p.m.f. of (X, Y) is as follows:

X \ Y	−2	0	2
−1	0.1	0.2	0.1
0	0.2	0.1	0.1
1	0.1	0.1	0

Find: **i)** F(0, 0) **ii)** $P(Y = 2 \mid X = 1)$ **iii)** $P(X^2 + Y^2 \leq 3)$.

c) Following is the joint p.m.f. of (X, Y)

X \ Y	0	1	2	3
0	0.1	0.05	0	0.20
1	0.15	0.25	0.05	0.05
2	0.05	0.10	0	0

Find:

i) Marginal distribution of X and Y.
ii) Conditional probability distribution of X given Y = 1.
iii) Conditional probability distribution of Y given X = 0.

P.T.O.

Q.3 Attempt **ANY TWO** of the following: **[12]**

a) Suppose X and Y are two discrete r.v.s. with joint probability distribution on $\{(x_i, y_j, p_{ij}), I = 1, 2, \ldots m, j = 1, 2, \ldots n\}$. prove that :

i) $E(X + Y) = E(X) + E(Y)$

ii) $E(XY) = E(X)\,E(Y)$, if X and Y are independent.

b) Define conditional variance of X given $Y = y_j$ and conditional variance of Y given $X = x_i$.

c) The joint p.m.f. of X and Y is given below:

X \ Y	1	2	3
0	0.1	0.2	0.3
1	0.1	0.2	0.2

Obtain: **i)** E (2X – Y) **ii)** E (XY) **iii)** E (X, Y)

Q.4 Attempt **ANY THREE** of the following: **[12]**

a) For (X, Y) a bivariate discrete r.v.s. $\sigma_x^2 = 9, \sigma_y^2 = 4$, cov (x, y) = 4. Find Var (2X – 3Y), Cov (2X, 3Y) and

b) Suppose X and Y are two discrete r.v.s. prove that: $Var(aX - bY) = a^2\ Var(X) + b^2\ Var(Y) - 2ab\ Cov(X, Y)$.

c) If the probability that a certain test yields a positive reaction is equal to 0.4. What is the probability that less than 4 negative reaction occur before the first positive one?

d) The joint p.m.f. of (X, Y) is given by

$p(x, y) = k(x^2 + y^2),\ k > 0,\ x = -1, 1,\ y = -2, 2$

$= 0 \qquad ,\quad$ otherwise

i) Obtain k **ii)** Are X and Y are independent?

Q.5 Attempt **ANY FOUR** of the following: **[12]**

a) Define joint probability mass function of (X, Y).

b) Define correlation coefficient ρ between two discrete r.v.s. X and Y. Give interpretation of the various values of ρ.

c) Let X and Y be two independent Poisson random variable with means 2 and 3 respectively. Find mean and variance of (X + Y).

d) State three real life situations in which geometric distribution can be applied.

e) Let (X, Y) be a variate discrete r.v.s. with joint p.m.f.

(x, y)	(0, 0)	(1, 0)	(0, 1)	(1, 1)
P (x, y)	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{1}{4}$

Find mean of X and Y.

f) A Poisson variate X has maximum probability at X = 4 and X = 5. Find variance and fourth cumulant of X.

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