BACHELOR OF SCIENCE (CBCS-2018 COURSE)
F. Y. B. Sc. Sem-I : WINTER :- 2021
SUBJECT: MATHEMATICS : CALCULUS

Day : Saturday
Date 29-01-2022

W-18308-2021

Time : 10:00 AM-01:00 PM
Max. Marks: 60

N.B.: 1) All questions are **COMPULSORY**.
2) Figures to the **RIGHT** indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.

Q.1 Attempt **ANY TWO** of the following : **(12)**

a) Prove that every continuous function on closed and bounded interval attains its bounds.

b) Examine the continuity of following function, where

$$f(x) = \frac{x^2}{4} - 4 \quad , 0 < x < 4$$
$$= 2 \quad , x = 4$$
$$= 4 - \frac{64}{x^2} \quad , x > 4 .$$

c) If $y = (\sin^{-1} x)^2$, then prove that $(1-x^2)y_{n+2} - (2n+1)x\, y_{n+1} - n^2 y_n = 0$.

Q.2 Attempt **ANY TWO** of the following : **(12)**

a) Show that a sequence $\{S_n\}$, where $S_n = \left(1 + \frac{1}{n}\right)^n$ is monotonic and bounded.

b) Find the expansion of $\tan x$ upto the terms in x^5.

c) Discuss the convergence of following series by using comparison test $\sum \frac{5\sqrt{n} - 1}{2n^2 + 3n + 2}$.

Q.3 Attempt **ANY TWO** of the following : **(12)**

a) State and prove Cauchy's mean value theorem.

b) Prove that $\frac{\pi}{6} + \frac{\sqrt{3}}{15} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$.

c) Verify Rolle's theorem for the function $f(x) = e^x(\sin x - \cos x)$, over $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

Q.4 Attempt **ANY THREE** of the following : **(12)**

a) Prove that $e^x \cos x = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} - \frac{x^5}{30} + \ldots\ldots\ldots\ldots$

b) Evaluate : $\lim_{x \to 2}\left(\frac{1}{x-2} - \frac{1}{\log(x-1)}\right)$.

c) Discuss the continuity of the function $f(x) = \sqrt{\frac{x-1}{x+3}}$.

d) Show that a function $f(x) = |x|$ is continuous everywhere but not differentiable

Q.5 Attempt **ANY FOUR** of the following : **(12)**

a) Define : i) suprimum ii) infimum .

b) Discuss the continuity of the function $f(x) = \frac{1}{1 - e^{\frac{1}{x}}}$ where $x \neq 0$ and $f(0) = 0$.

c) If $y = \cos(ax + b)$ then find y_n .

d) Evaluate : $\lim_{x \to 0} x^x$.

e) Define : i) convergent sequence ii) divergent sequence.

f) Discuss the convergence of series $\sum \frac{n}{2^n}$.

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