BACHELOR OF SCIENCE (CBCS-2018 COURSE)
F. Y. B. Sc. Sem-I : WINTER :- 2021
SUBJECT: MATHEMATICS : ALGEBRA

Day : Thursday
Date 27-01-2022 **W-18307-2021**
Time : 10:00 AM-01:00 PM
Max. Marks: 60

N. B. :

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q. 1 Attempt **ANY TWO** of the following: **(12)**

a) Prove that if A is a square matrix of order n, then the matrices A and adj A commute and the product is the scalar matrix $|A| I$.

$$i.e.\ A\,(adj\ A) = (adj\ A)\,A = |A| I.$$

b) Find the non-singular matrices P and Q such that PAQ is the normal form and hence find the rank of matrix A, where

$$A = \begin{bmatrix} 2 & 1 & 3 & -2 \\ 3 & -1 & 0 & 4 \\ 1 & 5 & 9 & 14 \end{bmatrix}$$

c) Verify Cayley-Hamilton theorem for the matrix,

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Q. 2 Attempt **ANY TWO** of the following: **(12)**

a) Prove that any two non-zero integers a and b have an unique (positive) g.c.d. and can be expressed in the form, d = (a, b) = ma + nb, for some $m, n \in \mathbb{Z}$.

b) Show that integers 1357 and 1166 are relatively prime. Find integers m and n such that 1 = 1357 m + 1166 n.

c) Solve the equations:

$$2x + 3y + 4z + u = 0$$
$$4x - 2y + z - 6u = 0$$
$$6x + 5y + 3z - u = 0.$$

Find a fundamental set of solutions.

Q. 3 Attempt **ANY TWO** of the following: **(12)**

a) State and prove De Moivre's theorem for positive and negative integers.

b) Express $\cos^7 \theta$ in terms of the cosines of multiple angles.

c) Solve the equation $x^7 + 1 = 0$ by using De Moivre's theorem.

P. T. O.

Q. 4 Attempt **ANY THREE** of the following: **(12)**

a) Find the eigen values of the matrix

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 8 & -8 & 4 \end{bmatrix}$$

b) If $A = \begin{bmatrix} x-3 & 1 & 3 \\ 0 & x & 9 \\ -3 & 3 & x \end{bmatrix}$, prove that $\forall x \in \mathbb{R} - \left\{3, \pm 3\sqrt{2}\right\}$ $\rho(A) = 3$.

c) Find the values of $\left(1 + i\sqrt{3}\right)^{10} + \left(1 - i\sqrt{3}\right)^{10}$.

d) Define equivalence relation. Show that R is an equivalence relation defined on a set of positive integers such that for all $x, y \in \mathbb{Z}^+$, xRy *iff* $x + y$ is an even number.

Q. 5 Attempt **ANY FOUR** of the following: **(12)**

a) If $A = \begin{bmatrix} 4 & -1 & 3 \\ 0 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, find adj A.

b) Define:

i) Inverse of a square matrix

ii) Rank of a matrix

c) Let $a, b, c, d \in \mathbb{Z}$ and if $a \equiv b \pmod{n}$, $c \equiv d \pmod{n}$ then show that $(a + c) \equiv (b + d) \pmod{n}$.

d) If p is prime and a, b are integers such that $p \mid ab$, then prove that either $p \mid a$ or $p \mid b$.

e) Find modulus and argument of,

$$z = \frac{i - i^2 + i^3 + i^4}{i^5 + i^6 + 2}.$$

f) If $z_1, z_2 \in \mathbb{C}$ then prove that

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \text{ and } \arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2.$$

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