B.TECH SEM – IV (2007 COURSE) (MECHANICAL ENGG.)/ (PRODUCATION ENGG.) : WINTER - 2017

SUBJECT: ENGINEERING MATHEMATICS - III

Day: **Monday**
Date: **20/11/2017**

W-2017-2425

Time: **02.30 PM TO 05.30 PM**
Max. Marks: 80

N.B.:

1) **Q. No. 1** and **Q. No. 5** are **COMPULSORY**. Out of the remaining attempt any **TWO** questions from each section.
2) Figures to the right indicate **FULL** marks.
3) Answers to both the sections should be written in **SEPARATE** answer book.
4) Use of non -programmable **CALCULATOR** is allowed.
5) Assume suitable data if necessary.
6) Draw neat labelled diagram **WHEREVER** necessary.

SECTION-I

Q.1 **a)** Solve $\frac{dx}{z(x+y)}=\frac{dy}{z(x-y)}=\frac{dz}{x^2+y^2}$. **(04)**

b) A circuit consists of an inductance L and condenser of capacity C in series. An alternating emf Esinnt is applied to it at time t = 0, the initial current and change on the condenser being zero, find the current flowing in the circuit at any time for $\omega = n$ where $\omega = \frac{1}{\sqrt{LC}}$. **(05)**

c) Suppose that the function $y(t)$ satisfies the differential equation: $y''(t)+2y'(t)+y(t)=3te^{-t}$, $t>0$ with initial values $y(0)=4$, $y'(0)=2$. Find the Laplace transform of $y(t)$. **(05)**

Q.2 Solve any **THREE**: **(13)**

a) $(D^2-4D+3)y = x^3e^{2x}$

b) $(D^2+3D+2)y = e^{e^x}+\cos e^x$

c) $(D^4-1)y = \cos x \cosh x$

d) $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=\cos(\log x)+x\sin(\log x)$

e) $\frac{d^2y}{dx^2}+y=\cos ecx$

P. T. O.

Q.3 a) An elastic string is stretched between two fixed points at a distance l apart, one end is taken at the origin and at a distance $\frac{2l}{3}$ from this end the string is displaced a distance 'a' transversely and is released from rest when in this position. Find $y(x,t)$, if y satisfies the equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. **(07)**

b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ *if* **(06)**

i) u is finite $\forall t$

ii) $u = 0$ when $x = 0, \pi$ $\forall t$

iii) $u = \pi x - x^2$ when $t = 0$ and $0 \le x \le \pi$.

Q.4 a) Find Fourier sine transform of $e^{-|x|}$. Hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx$. **(04)**

b) Find Fourier cosine transform of $\frac{1}{1+x^2}$ **(04)**

c) Find the inverse Laplace transform of $\frac{2s+1}{(s^2+s+1)^2}$. **(05)**

SECTION-II

Q.5 a) Mean and variance of binomial distribution are 6 and 2 respectively. Find $p(r \ge 1)$. **(04)**

b) Find the tangential and normal components of acceleration at time t =1 for the curve $\bar{r} = t^2 \bar{i} - t^3 \bar{j} + t^4 \bar{k}$. **(05)**

c) An urn contains 6 white and 8 red balls, second urn contains 9 white and 10 red balls. One ball is drawn at random from the first urn and put into the second urn without noticing its colour. A ball is then drawn at random from the second urn. What is the probability that it is red? **(05)**

Q.6 a) If the two lines of regression are $9x + y - \lambda = 0$ and $4x + y = \mu$ and means of x and y are 2 and -3 respectively. Find the values of λ, μ and the correlation coefficient between x and y. **(04)**

b) Find probability that almost 5 defective fuses will be found in a box of 200 fuses if 2% of such fuses are defective. **(05)**

c) Find a, b so that the surface $ax^2 - 2byz = (a+4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at $(1,-1,2)$. **(04)**

Q.7 a) Find directional derivative of $\phi = e^{2x}\cos yz$ at the origin in the direction tangent to the curve **(04)**

$x = a\sin t,\ y = a\cos t,\ z = at$ at $t = \frac{\pi}{4}$.

b) Show that $\overline{F} = r^2\overline{r}$ is conservative and obtain the scalar potential associated with it. **(04)**

c) Find the work done in moving a particle along **(05)**

$x = a\cos\theta, y = a\sin\theta,\ z = b\theta$ from $\theta = \frac{\pi}{4}$ to $\theta = \frac{\pi}{2}$ under a field of force given by

$\overline{F} = -3a\sin^2\theta\cos\theta\,\overline{i} + a\left(2\sin\theta - 3\sin^3\theta\right)\overline{j} + b\sin 2\theta\,\overline{k}$.

Q.8 a) Verify Green's theorem for **(07)**

$\overline{F} = x\overline{i} + y^2\overline{j}$ over the first quadrant of the circle $x^2 + y^2 = 1$.

b) Verify Divergence theorem for **(06)**

$\overline{F} = 2x^2y\overline{i} - y^2\overline{j} + 4xz^2\overline{k}$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and $x = 2$.

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