B.TECH SEM - IV (2007 COURSE) (CHEMICAL ENGG.) :
WINTER - 2017
SUBJECT : ENGINEERING MATHEMATICS- III

Day : **Monday**
Date : **20/11/2017**

W-2017-2395

Time **02.30 PM TO 05.30 PM**
Max. Marks : 80

N.B.

1) Q.1 and Q.5 are **COMPULSORY.** Out of the remaining attempt any **TWO** questions from each Section.
2) Figures to the right indicate **FULL** marks.
3) Answers to both the sections should be written in **SEPARATE** answer book.
4) Use of non-programmable calculator is allowed.

Q.1 a) Solve **(04)**

$$\frac{dx}{2x}=\frac{dy}{-y}=\frac{dz}{4xy^2-2z}$$

b) For a steady flow through a hollow cylindrical stream pipe whose inner and outer radii are r_1 and r_2 respectively, the temperature u at a radial distance r $(r_1<r<r_2)$ from the axis of cylinder is given by **(05)**

$$r\frac{d^2u}{dr^2}+\frac{du}{dr}=0.$$

If u_1, u_2 are respectively the temperatures at inner and outer surfaces of the pipe, find u as function of r.

c) Show that the Fourier transform of $f(x)=e^{\frac{-x^2}{2}}$ is $e^{\frac{-\lambda^2}{2}}$. **(05)**

Q.2 a) Solve any **TWO:** **(08)**

i) $\left(D^4+m^4\right)y=\sin mx$

ii) $\left(D^2-1\right)y=x\sin x+\left(1+x^2\right)e^x$

iii) $(D^2-D-2)y=2\log x+\frac{1}{x}+\frac{1}{x^2}$

b) Solve by method of variation of parameters **(05)**

$\left(D^2-2D+2\right)y=e^x\tan x$

Q.3 a) The deflection of a strut with one end built in and the other supported and subject to end thrust p, satisfies the equation, **(07)**

$$\frac{d^2y}{dx^2}+a^2y=\frac{a^2R}{P}(1-x)$$

Given that $\frac{dy}{dx}=y=0,$ when $x=0$ and $y=0$ when $x=l,$ prove that

$$y=\frac{R}{a}\left[\frac{\sin ax}{a}-\cos ax+l-x\right] \text{ where } al=\tan al.$$

b) Solve $\frac{\partial u}{\partial t}=c^2\frac{\partial^2u}{\partial x^2}$ it **(06)**

i) u is finite, $\forall t$

ii) $u(0,t)=0, \forall t$

iii) $u(l,t)=0, \forall t$

iv) $u(x,0)=u_0$ for $0\le x\le l$

whose l being length of bar.

P.T.O.

Q.4 a) Find the Fourier transform of **(07)**

$f(x) = 1, |x| > 1$

$= 0, |x| > 1$

Hence evaluate

$$\int_0^\infty \frac{\sin x \cos \lambda x}{x} dx$$

b) Find the sine transform of $f(x) = \frac{e^{-ax}}{x}$ $0 \le x < \infty$ and use it evaluate **(06)**

$$\int_0^\infty \tan^{-1}\frac{x}{a} \sin x dx$$

SECTION – II

Q.5 a) Find the work done in force field **(05)**
$\overline{F} = 3x^2 yi + (x^3 + 2yz)j + y^2 k$ in moving an object from the point $(1,-2,1)$ to $(3,1,4)$.

b) Solve the following differential equation by Laplace transform method **(05)**

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-x}; y(0) = y'(0) = 1.$$

c) A particle moves along the curve $\overline{r} = t^2\hat{i} - t^3\hat{j} + t^4\hat{k}$. Find magnitude of tangential and normal components of acceleration at $t = 1$. **(04)**

Q.6 a) Find the Laplace transform of $F(t) = \int_0^\infty \frac{e^{-t} - e^{-3t}}{4} dt$. **(04)**

b) Find the Laplace-Inverse transform of $f(s) = \cot^{-1}(s+1)$. **(04)**

c) Find the Laplace transform of $f(t) = \begin{cases} t\ ; 0 < t < 4 \\ 5\ ;\ t > 4 \end{cases}$. **(05)**

Q.7 a) Find the directional derivative of $\phi = x^2 y + xyz + z^3$ at $(1,2,-1)$ along the normal to the surface $x^3 y^3 - 4xy - y^2 z = 0$ at $(1,2,0)$ **(05)**

b) Show that $(\overline{a} \cdot \nabla)\overline{r} = \overline{a}$ where $\overline{r} = xi + yj + zk$ and $\overline{a}$ is a constant vector. **(03)**

c) Find the angle between the tangents to the curve **(05)**
$x = t,\ y = t^2, z = t^3$ at $t = 1$ and $t = -1$.

Q.8 a) Show that $\overline{F} = (2xz^3 + 6y)i + (6x - 2yz)j + (3x^2 y^2 - y^3)k$ is irrotational. Find the corresponding scalar potential ϕ such that $\overline{F} = \nabla\phi$. **(05)**

b) Prove that $\int_C (\overline{a} \times \overline{r}) \cdot dr = 2\overline{a} \iint_S d\overline{S}$ where $\overline{r} = xi + yj + zk$ and $\overline{a}$ is a constant vector. **(04)**

c) Show that the volume enclosed by a closed surface is $\frac{1}{6}\iint \nabla r^2 \cdot \overline{dS}$. **(04)**

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