B.TECH. SEM -I (CHEMICAL/ CIVIL/ ELECTRICAL/ MECHANICAL/ PRODUCTION/ COMPUTER/ INFO. TECH./ ELECTRONICS / BIO MEDICAL / E & TC) 2014 COURSE (CBCS) : WINTER - 2017

SUBJECT : ENGINEERING MATHEMATICS – I

Day : **Thursday** Time : **10.00 AM TO 01.00 PM**

Date : **11/01/2018** Max. Marks : 60

W-2017-1996

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.
4) Assume suitable data if necessary.

Q.1 a) Reduce into normal form and find its rank. **[05]**

$$\begin{bmatrix} 3 & -6 & 4 & -3 & 2 \\ 2 & -4 & 3 & 1 & 0 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$$

b) Examine for linear dependence or independence of the set of vectors. If dependent, find the relation between them. **[05]**

$[1 \quad 2 \quad 4], [2 \quad -1 \quad 3], [0 \quad 1 \quad 2], [-3 \quad 7 \quad 2]$.

OR

Find Eigen values and Eigen vectors for the following matrix. **[10]**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Q.2 a) Evaluate : $(1+i)^{100} + (1-i)^{100}$ **[05]**

b) A square lies above real axis in Argand's diagram and has two of its vertices at origin and the point 3 + 2i. Find the remaining two vertices of the square. **[05]**

OR

If $\alpha + i\beta = \tanh\left(x + i\,\frac{\pi}{4}\right)$, then prove that $\alpha^2 + \beta^2 = 1$. **[10]**

Q.3 If $x = \cos\left(\log y^{\frac{1}{m}}\right)$ then prove that **[10]**

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (m^2+n^2)y_n = 0$.

OR

If $y = \sin^{-1}x$, then obtain $y_n(0)$. **[10]**

P.T.O.

Q.4 a) Test the convergence of the series $1+\frac{1}{2}+\frac{1.3}{2.4}+\frac{1.3.5}{2.4.6}+\ldots\ldots$ **[05]**

b) Evaluate : $\lim_{x\to 0}\left[\frac{a}{x}-\cot\left(\frac{x}{a}\right)\right]$. **[05]**

OR

a) Test the convergence of the series $\sum_{1}^{\infty}\frac{1^2.5^2.9^2\ldots\ldots(4n-3)^2}{4^2.8^2.12^2\ldots\ldots(4n)^2}$. **[05]**

b) Find values of a, b if $\lim_{x\to 0}\frac{a\cos x-a+bx^2}{x^4}=\frac{1}{12}$. **[05]**

Q.5 If $z=f(u,v)$ and $u=xcost-ysint,\ v=xsint+ycost$, where t is a constant, then show that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=u\frac{\partial z}{\partial u}+v\frac{\partial z}{\partial v}$ **[10]**

OR

If $u=\frac{x^3+y^3}{y\sqrt{x}}+\frac{1}{x^7}\sin^{-1}\left(\frac{x^2+y^2}{2xy}\right)$ then find the value of $x^2u_{xx}+2xyu_{xy}+y^2u_{yy}$ at $(1,1)$. **[10]**

Q.6 a) If $u=x+y+z,\quad uv=y+z,\quad uvw=z,$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. **[05]**

b) If $x+y=ze^{\theta}\cos\phi,\ x-y=zie^{\theta}\sin\phi$, then prove that JJ′ = 1. **[05]**

OR

Find the maximum and minimum values of $f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$. **[10]**

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