B.TECH. SEM -I (2007 COURSE) (ALL BRANCHES) : WINTER - 2017

SUBJECT: ENGINEERING MATHEMATICS - I

Day: **Thursday** Time: **10.00 AM TO 01.00 PM**

Date: **11/01/2018** **W-2017-2342** Max. Marks: 80

N.B.;

1) **Q. No. 1** and **Q. No. 5** are **COMPULSORY**. Out of remaining questions attempt **ANY TWO** questions from each section.
2) Answer to both the sections should be written in **SEPARATE** answer books.
3) Draw neat and labeled diagram **WHEREVER** necessary.
4) Figures to the right indicate **FULL** marks.
5) Assume suitable data, if necessary.

SECTION - I

Q.1 a) Define rank of matrix. Find the rank of A where, **(05)**

$$A=\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

b) Prove that : $\dfrac{1+\cos\alpha + i\sin\alpha}{1-\cos\alpha + i\sin\alpha} = \cot(\dfrac{\alpha}{2})\, e^{i\left(\alpha-\frac{\pi}{2}\right)}$ **(05)**

c) Find nth derivative of : $\dfrac{x}{(x+1)^4}$ **(04)**

Q.2 a) Investigate for what values of λ and μ, the system of simultaneous equations: **(06)**

$x+y+z=6$

$x+2y+3z=10$

$x+2y+\lambda z=\mu$

have **i)** no solution **ii)** a unique solution **iii)** an infinite number of solutions.

b) Examine for linear dependence or independence the following system of vectors. If dependent, find relation between them. **(04)**

$x_1=(1, 1, 1, 3),\ x_2=(1, 2, 3, 4),\ x_3=(2, 3, 4, 7)$

c) Show that $A=\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & {} & \cos\theta \end{bmatrix}$ *is* orthogonal. **(03)**

Q.3 a) On Argand diagram, the circumcentre of an equilateral triangle represents the complex number 1 + i. If one vertex represents the complex number -1 + 3i, find the complex number represented by the other two vertices. **(04)**

b) Solve : $x^4-x^3+x^2-x+1=0$. **(05)**

c) If $\tan(x+iy)=i$, where x and y are real, prove that x is indeterminate and y is infinite. **(04)**

P.T.O.

Q.4 a) If $y=\left(x+\sqrt{x^2-1}\right)^m$, show that $\left(x^2-1\right)y_{n+2}+(2n+1)\,xy_{n+1}+\left(n^2-m^2\right)y_n=0$. **(04)**

b) If $I_n=\dfrac{d^n}{dx^n}\left(x^n\log x\right)$, prove that $I_n=nI_{n-1}+(n-1)!$ **(04)**

c) Considering function $f(x)=\sqrt{x}$ and $g(x)=1/\sqrt{x}$, prove that 'c' of Cauchy mean value theorem is geometric mean between a and b. **(05)**

SECTION - II

Q.5 a) Using Taylor's theorem, express $7+(x+2)+3(x+2)^3+(x+2)^4$ in ascending powers of x. **(04)**

b) If $u=\sin^{-1}\left\{\dfrac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}\right\}^{1/2}$ show that **(05)**

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{144}\left(13+\tan^2 u\right).$$

c) Examine for the functional dependence **(05)**

$u=\sin^{-1}x+\sin^{-1}y,\ v=x\sqrt{1-y^2}+y\sqrt{1-x^2}$

Q.6 a) Test the convergence of the series **(04)**

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\ldots\ldots$$

b) Test for convergence the series whose n^th^ term is $\dfrac{n^3}{(n-1)}$. **(04)**

c) Expand $\log\tan\left(\dfrac{\pi}{4}+x\right)$ in ascending powers of x. **(05)**

Q.7

a) Find a and b if $\lim\limits_{x\to 0}\dfrac{x(1+a\cos x)-b\sin x}{x^3}=1$ **(05)**

b) If $x=\dfrac{\cos\theta}{u}$, $y=\dfrac{\sin\theta}{u}$, evaluate $\left(\dfrac{\partial x}{\partial u}\right)_\theta\cdot\left(\dfrac{\partial u}{\partial x}\right)_y+\left(\dfrac{\partial y}{\partial u}\right)_\theta\cdot\left(\dfrac{\partial u}{\partial y}\right)_x$ **(05)**

c) If $(\cos x)^y=(\sin y)^x$ then find $\dfrac{dy}{dx}$. **(03)**

Q.8 a) For the transformation $x=e^u\cos v,\ y=e^u\sin v$ prove that $\dfrac{\partial(x,y)}{\partial(u,v)}\cdot\dfrac{\partial(u,v)}{\partial(x,y)}=1$. **(05)**

b) The area of rectangular field is calculated by measuring its length and breadth. If there is an error of 2% in measuring the length and an error of 3% in measuring the breadth of the field, find the approximate % error in the calculated area of field. **(04)**

c) Locate the stationary points of $x^4+y^4-2x^2+4xy-2y^2$ and determine their nature. **(04)**

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