B.TECH SEM - III (2007 COURSE) (INF. TECH.) : WINTER - 2017

SUBJECT: DISCRETE MATHEMATICS

Day : **Wednesday** Time : **10.00 AM TO 01.00 PM**

Date : **17/01/2018** **W-2017-2376** Max. Marks : 80

N.B.:

1) **Q. No. 1** and **Q.No.5** are **COMPULSORY**. Out of the remaining questions attempt **ANY TWO** questions from each section.
2) Answers to both the sections should be written in the **SEPARATE** answer books.
3) Use of non programmable **CALCULATOR** is allowed.
4) Figures to the right indicate **FULL** marks.

SECTION – I

Q.1 **a)** Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences (without using truth table). **[05]**

b) If f and g are bijections on a set A. Prove that $f \circ g$ is also a bijection. **[05]**

c) What is well ordering? How it is related to strong induction. **[04]**

Q.2 **a)** Show that if A and B are sets then: **[07]**

i) $A \oplus B = B \oplus A$

ii) $A \oplus B = (A \cup B) - (A \cap B)$

iii) $(A \oplus B) \oplus = A$

b) **i)** Prove that : $(\forall x)(p(x) \vee q(x)) \Rightarrow (\forall x)\, p(x) \vee (\exists x) q(x)$. **[06]**

ii) obtain CNF form of $(p \vee \bar{q}) \rightarrow q$.

Q.3 **a)** Solve the recurrence relation $S(n) = S(n-1) + 2(n-1)$ with $S(0) = 3$, $S(1) = 1$. **[07]** by finding its generating function.

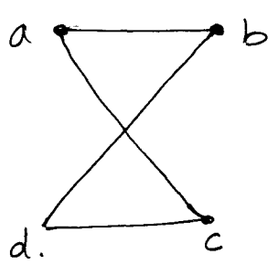
b) If R denotes the set of real number and $f : R \rightarrow R$ is given by $f(x) = x^3 - 2$ find f^{-1}. **[06]**

Q.4 **a)** Using mathematical induction prove that $5^n - 1$ is divisible by 4 for $n \geq 1$. **[07]**

b) Use mathematical induction to prove the following inequality $n < 2^n$ for all positive integers. **[06]**

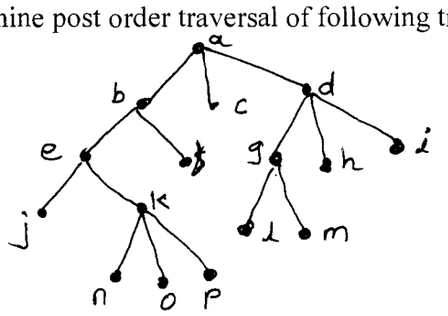
SECTION – II

Q.5 **a)** How many paths of length 4 are there from a to d in following simple graph? **[05]**



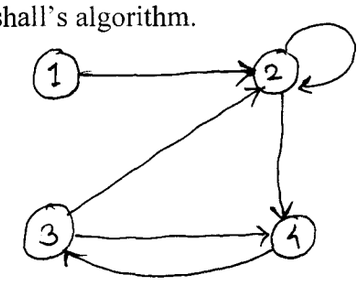
P.T.O.

b) Determine post order traversal of following tree T. **[05]**

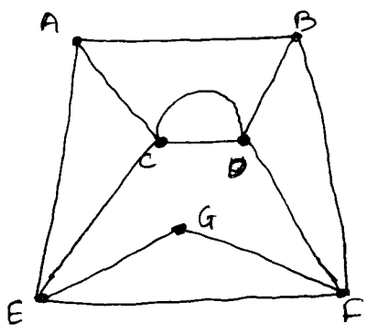


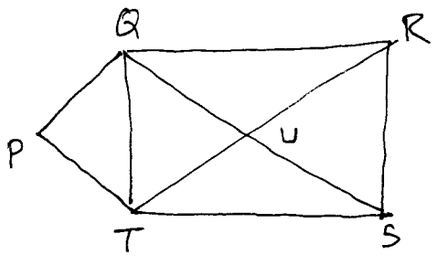
c) Define sub group and normal sub group with example. **[04]**

Q.6 **a)** Find transitive closure of the relation R on set A defined by the given digraph using Warshall's algorithm. **[07]**

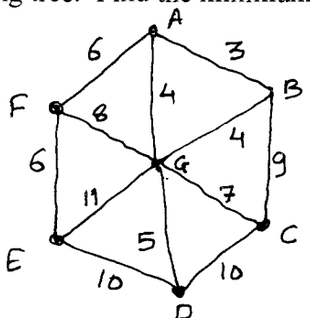


b) Determine the Eulerion path and Hamiltonian path, if exist, in the following graph. **[06]**





Q.7 **a)** What do you mean by spanning tree? State the prims algorithm to find the minimum spanning tree. Find the minimum spanning tree of the given graph G. **[07]**



b) What is optimal binary tree? State and explain Huffman algorithm to find the optimal binary tree. Construct an optimal tree for weights: 8, 9, 10, 11, 13, 15, 22. **[06]**

Q.8 **a)** In any Ring $(R+\cdot)$ prove that: **[07]**

i) The zero element z is unique.

ii) The additive inverse of each ring element is unique.

b) Let $(A, *)$ be an algebric system such that for all **[06]**

a, b $\in$ A $(a*b)*a = a\,(a*b)*b = (b*a)*a$.

Show that

$a*(a*b) = a*b$ for all $a, b \in A$ and $a*a = (a*b)*(a*b)$ for all $a, b \in A$

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