M. TECH.-I (CIVIL-HYDRAULIC ENGINEERING) (CBCS – 2015 COURSE) : WINTER - 2017

SUBJECT : COMPUTATIONAL METHODS IN HYDRAULIC ENGINEERING

Day **Friday** Time **11.00 AM TO 02.00 PM**

Date **19/01/2018** **W-2017-2774** Max. Marks : 60

N.B.

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.
3) Answers to both the sections should be written in **SEPARATE** answer book.
4) Use of non-programmable calculator is allowed.

SECTION – I

Q.1 **a)** If $f(z)$ is an analytic function of z, prove that **(05)**

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2 .$$

b) Find analytic function $f(z) = u + iv$, given $u = a(1+\cos\theta)$. **(05)**

OR

Prove that the necessary and sufficient conditions for the derivative of the function $w = u(x,y) + iv(x,y) = f(z)$ to exist for all values of z in a region R, are **(10)**

i) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ are continuous function of x and y in R.

ii) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

Q.2 **a)** Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. **(05)**

b) Evaluate $\int_C \tan z dz$ where C is the circle |z|=2. **(05)**

OR

a) Find the transformation which will map the interior of the infinite strip bounded by the lines $v = 0, v = \pi$ onto the upper half of the z-plane. **(05)**

b) Show that $\oint_C (z+1)dz = 0$, where C is the boundary of the square whose vertices are at the points $z = 0, z = 1, z = 1+i$ and $z = i$. **(05)**

Q.3 Solve the partial differential equation $\nabla^2 u = -10(x^2+y^2+10)$ over the square with sides $x = 0 = y, x = 3 = y$ with $u = 0$ on the boundary and mesh length =1. **(10)**

OR

a) Classify the following equations: **(04)**

i) $y^2u_{xx} - 2yu_{xy} + u_{yy} - u_y = 8y.$

ii) $y^2u_{xx} + u_{yy} + u^2{}_x + u^2{}_y + 7 = 0$

b) Solve by Relaxation method, the equations: **(06)**

$10x - 2y - 3z = 205$

$-2x + 10y - 2z = 154$

$-2x - y + 10z = 120$

SECTION – II

Q.4 **a)** Fit the exponential curve $y = ae^{bx}$ to the following data: **(05)**

x:	2	4	6	8
y :	25	38	56	84

b) A solid of revolution is formed by rotating about the x-axis, the area between x-axis the lines $x = 0$ and $x = 1$ and a curve through the points with following co-ordinates. **(05)**

x:	0.00	0.25	0.50	0.75	1.00
y :	1.0000	0.9896	0.9586	0.9089	0.8415

Estimate the volume of the solid formed using Simpon's 1/3rd rule.

OR

a) Fit a second degree parabola to the following data: **(05)**

x:	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y :	1.1	1.3	1.6	2.0	2.7	3.4	4.1

b) Given that **(05)**

x:	4.0	4.2	4.4	4.6	4.8	5.0	5.2
logx:	1.3863	1.4351	1.4816	1.5261	1.5816	1.6094	1.6484

Evaluate $\int_4^{5.2} \log x \, dx$ by Trapezoidal rule.

Q.5 **a)** Two lines of regression are given by **(06)**

$5y - 8x + 17 = 0$ and $2y - 5x + 14 = 0$

If $\sigma_y^2 = 16$, find:

i) the mean value of x and y **ii)** σ_x^2

iii) the coefficient of correlation between x and y.

b) If $r_{12} = 0.6$, $r_{23} = 0.35$, and $r_{31} = 0.4$, then find $R_{3.12}$. **(04)**

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OR

a) Calculate the first four moments about mean for the following data: **(05)**

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	1	6	10	15	11	7

Also calculate β_1 and β_2.

b) Find rank correlation coefficient to the following data: **(05)**

x:	65	63	67	64	68	62	70	66	68	67	69	71
y:	68	66	68	65	69	66	68	65	71	67	68	70

Q.6 **a)** There are three bags : first containing 1 white, 2 red, 3 green balls; second 2 white, 3 red, 1 green balls and third 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be one white and one red. Find the probability that the balls so drawn came from the second bag. **(05)**

b) Find mean and variance of Poisson's distribution **(05)**

OR

a) The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If six bombs are dropped, find the probability that **(05)**

i) exactly two will strike the target.

ii) atleast two will strike the target.

b) A survey of 320 families with 5 children is given below: **(05)**

No. of boys	5	4	5	2	1	0	Total
No. of girls	0	1	2	3	4	5	
No. of families	14	56	110	88	40	12	320

Is this result consistent with hypothesis that the male and female birth are equally possible.

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