

Day **Thursday**
Date **02/11/2017**

W-2017-0573

Time: **11.00 A.M. TO 02.00 PM**
Max. Marks: 60

N. B. :

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.
- 3) Neat diagrams must be drawn **WHEREVER** necessary.
- 4) Use of **logarithmic table / calculator** is allowed.
- 5) All the symbols have their usual meaning unless otherwise stated.

Q.1 Answer **ANY TWO** of the following : **(12)**

- a) Find the approximate value $\sqrt{(2.99)^2 + (3.99)^2}$ Using method of differential.
- b) Determine the value of $(1+i)^8 + (1-i)^8$
- c) Find the projection of vector $\vec{B} = \hat{i} + 5\hat{j} + 3\hat{k}$ on the vector $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

Q.2 Attempt **ANY TWO** of the following: **(12)**

- a) i) If $F = \sqrt{x^2 + y^2}$, $x = p \cos \theta$ and $y = q \sin \theta$ obtain the expression for $\frac{\partial F}{\partial \theta}$ by using chain rule.
ii) show that $|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 = |\vec{A}|^2 |\vec{B}|^2$
- b) If $\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{B} = \hat{i} + 4\hat{j} - 2\hat{k}$ find
 - i) $\vec{A} \times \vec{B}$
 - ii) $\vec{B} \times \vec{A}$
 - iii) $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$
- c) Show that $\vec{\nabla} \times (\phi \vec{A}) = (\vec{\nabla} \phi) \times \vec{A} + \phi (\nabla \times \vec{A})$

Q.3 Attempt **ANY TWO** of the following: **(12)**

- a) Using $\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$ and $\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$ prove that
 - i) $\sin 2\theta = 2 \sin \theta \cdot \cos \theta$
 - ii) $\cos^2 \theta + \sin^2 \theta = 1$
- b) Prove that $\nabla^2 \left(\frac{1}{r} \right) = 0$
- c) Interpret the following equation geometrically $|z+3| = 2|z-3|$

Q.4 Attempt **ANY THREE** of the following: **(12)**

- a) If $\vec{A} = x^2z\vec{i} + xy^2z\vec{j} - 3yz^2\vec{k}$ determine $\text{curl } \vec{A}$ at the point (1, 1, 1).
- b) Find the area of triangle with vertices at (3, -1, 2) (1, -1, -3) and (4, -3, 1).
- c) Determine the value of x and y if $(x + iy) = (1 + i\sqrt{3})^4$
- d) If $F = x^3 + xy + y^3$ then find $\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial^2 F}{\partial x \partial y}, \frac{\partial^2 F}{\partial y \partial x}$

Q.5 Attempt **ANY FOUR** of the following: **(12)**

- a) Determine the value of $i + i^2 + i^3 + i^4$
- b) Show that $e^{\pi i} = -1$ and $e^{i\pi/2} = i$
- c) Prove that $\vec{A} = \hat{i} + 2\hat{j} + 8\hat{k}$ and $\vec{B} = 2\hat{i} + 3\hat{j} - \hat{k}$ are perpendicular to each other.
- d) Prove that vector $\vec{A} = 3yz\hat{i} + zx\hat{j} + 4xy\hat{k}$ is solenoidal.
- e) If $F = e^{xy}$ Calculate $\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial^2 F}{\partial y^2}$
- f) State degree and order of following differential equation
$$\frac{d^4 y}{dx^4} - \sqrt{y^4} - 5 = 0$$

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