S.Y.B.SC. SEM – IV (2014 COURSE) :WINTER - 2017

SUBJECT : MATHEMATICS : VECTOR CALCULUS (M-41)

Day : **Monday**
Date : **06/11/2017**

W-2017-0635

Time : **03.00 PM TO 05.00 PM**
Max. Marks : 40

N.B.

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable calculator is allowed.

Q.1 Attempt any **TWO** of the following: **(10)**

a) Prove that a differentiable vector function $\vec{u}(t)$ on $[a,b]$ is of constant magnitude if and only if $\vec{u}\bullet\frac{d\vec{u}}{dt}=0$.

b) If $\phi = xy + yz + zx$ and $\vec{u} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$, then find $\frac{\partial^2(\phi\vec{u})}{\partial z\,\partial x}$ *at* (3, 2, −4).

c) Find the scalar function $\phi(x, y, z)$ if $grad\,\phi = y(2zx-1)\hat{i} + x(xz-1)\hat{j} + (x^2y+4)\hat{k}$ and $\phi(2,1,-1)=0.$

Q.2 Attempt any **TWO** of the following: **(10)**

a) If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\bar{r}|$ then find :

i) ∇r **ii)** ii) $\mathrm{div}\,\bar{r}$ **iii)** grad (r^n) **iv)** $\mathrm{div}(r^n\bar{r})$ **v)** $\mathrm{curl}(r^n\bar{r})$.

b) Find the directional derivative of $\phi = 2x^3y - 3y^2z$ at $P((1,2,-1)$ in the direction towards $Q(3,-1,5)$

c) Evaluate $\iint_S \phi\bar{n}\,dS$, where $\phi = \frac{3xyz}{8}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.

Q.3 Attempt any **TWO** of the following: **(10)**

a) Let $\bar{f}$ be a continuously differentiable vector field. Then show that $\bar{f}$ is conservative if and only if it is the irrotational.

b) Using Green's theorem evaluate $\oint_C\left[(e^{-x}\sin y\,dx + e^{-x}\cos y\,dy\right]$, where C is the rectangle formed by $x=0, x=\pi, y=0, y=\frac{\pi}{2}$.

c) By using divergence theorem evaluate $\iint_S[(x+z)\,dy\,dz + (y+z)dz\,dx + (x+y)dx\,dy]$, where S is the surface of the sphere $x^2+y^2+z^2=9.$

P.T.O.

Q.4 Attempt any **FIVE** of the following: **(10)**

a) Eliminate $\overline{a}$ and $\overline{b}$ from $\overline{r} = \overline{a}\cos 2t + \overline{b}\sin 2t$ and obtain the differential equation.

b) Find the equations of the tangent plane to the surface $xy + yz + zx = 7$ at $(1,1,3)$

c) Find the unit vector to the surface $x^2 + y^2 - z = 1$ at $(1,1,1)$.

d) Prove that the vector $\overline{u} = 3y^4z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}$ is solenoidal.

e) The acceleration $\overline{a}$ of a particle at any time $t \geq 0$ is given by $\overline{a} = e^{-t}\hat{i} - 6(t+1)\hat{j} + 3\sin t\hat{k}$. Find the velocity $\overline{v}$ at $t = 0$.

f) Define divergence of vector point function.

g) State Stoke's theorem.

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