S.Y.B.SC. SEM – III (2014 COURSE) : WINTER - 2017

SUBJECT : MATHEMATICS : GROUP THEORY & DIFFERENTIAL EQUATIONS (M – 32)

Day : **Wednesday** Time : **12.00 NOON TO 02.00 PM**

Date : **08/11/2017** Max. Marks : 40

W-2017-0623

N.B.:

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **[10]**

a) Show that if a, b are any two elements in a group G then $O(a) = O(b^{-1}ab)$.

b) If in a group G, every element is its own inverse then prove that G is abelian.

c) Solve: $y + px = x^4 p^2$, where $p = \frac{dy}{dx}$.

Q.2 Attempt **ANY TWO** of the following: **[10]**

a) Define cyclic group. Give an example of a cyclic group. Show that every cyclic group is abelian.

b) Let A and B be subgroups of a group G such that $A \cup B$ is also a subgroup of G. Show that either $A \subseteq B$ or $B \subseteq A$.

c) Solve the differential equation $y = 2px + p^2 y$, where $p = \frac{dy}{dx}$.

Q.3 Attempt **ANY TWO** of the following: **[10]**

a) Show that $\frac{1}{f(D^2)} \cos(ax+b) = \frac{\cos(ax+b)}{f(-a^2)}$, where $f(-a^2) \neq 0$.

b) Solve : $(D^2 - 1)y = xe^{2x}$.

c) Solve : $(D^2 + 4D + 4)y = e^{-2x} + x^3$.

Q.4 Attempt **ANY FIVE** of the following: **[10]**

a) Solve: $p^2 - 7p + 12 = 0$, where $p = \frac{dy}{dx}$.

b) Show that the substitutions $x^2 = u$ and $y^2 = v$, converts the equation of the type $y^2 + pxy + f\left(\frac{py}{x}\right)$ into Clairaut's form.

c) Solve : $D^3 + 7D^2 + 16D + 10 = 0$.

d) Find the particular integral of $(D^3 + 3D^2 + 3D + 1)y = e^{-x}$.

e) Show that the intersection of two subgroups of a group is a subgroup.

f) Let S = {1, -1, i, -1} and (S, *) be a group, where * is multiplication of complex numbers. Find inverse of every element of S.

g) Let $Q_1 = Q - \{1\}$ and $(Q_1, *)$ be a group. A composition * on Q_1 is defined by $a * b = a + b - ab, \forall a, b \in Q_1$ then find identity element in Q_1 with respect to *.

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