S.Y.B.SC. SEM – IV (2014 COURSE) : WINTER - 2017

SUBJECT : MATHEMATICS : COMPLEX VARIABLES (M – 42)

Day : Wednesday
Date : 08/11/2017

W-2017-0637

Time : 03.00 PM TO 05.00 PM
Max. Marks : 40

N.B.:

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **[10]**

a) Show that a necessary condition that a function $f(z) = u(x, y) + iv(x, y)$ be analytic at a point $z = x + iy$ is that at (x, y) the real and imaginary parts u and v of $f(z)$ satisfy the Cauchy – Riemann equations $u_x = v_y$ and $u_y = -v_x$.

b) Find an analytic function $f(z) = u + iv$, where $v(x, y) = 3x^2y - 6xy + 3y - y^3$.

c) If $f(z)$ is an analytic function with constant amplitude, then prove that $f(z)$ is a constant function.

Q.2 Attempt **ANY TWO** of the following: **[10]**

a) Using Cauchy's theorem obtain the value of $\int_C e^z \, dz$, where C is the circle $|z| = 1$ and deduce that $\int_0^{2\pi} e^{\cos\theta} \left[\sin(\theta + \sin\theta)\right] d\theta = 0$.

b) Evaluate : $\int_C \frac{e^z}{(z+1)^2} dz$, where C is the circle $|z-1| = 3$, using Cauchy's integral formula.

c) Obtain expansion of $\frac{1}{z^2 - 3z + 2}$ for $0 < |z| < 1$.

Q.3 Attempt **ANY TWO** of the following: **[10]**

a) State and prove Cauchy's residue theorem.

b) Evaluate by contour integration $\int_C \frac{z+2}{z^2-1} dz$, where C is the circle $|z| = 2$ taken counter clockwise.

c) Evaluate by contour integration $\int_{-\infty}^{\infty} \frac{dx}{x^2+1}$.

Q.4 Attempt **ANY FIVE** of the following: **[10]**

a) Show that a function $f(z)$ of complex variables is differentiable at z_0 then it is continuous at z_0.

b) Evaluate $\lim_{z \to i} \frac{iz^3 - 1}{z + i}$.

c) Obtain Maclaurin's series for coshz.

d) Determine the poles and their order for the function $f(z) = \frac{z^3 + 1}{(z^2+3)(z^2-4)^3}$.

e) Find the residues of $f(z) = \frac{z^2}{(z-2)(z+3)^2}$ at the simple poles.

f) Define: **i)** Simple closed curve **ii)** Simply connected region.

g) Show that if $f(z)$ and $g(z)$ are continuous at z_0 then $f(z) \cdot g(z)$ is continuous at z_0.

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