S.Y.B.SC. SEM – III (2014 COURSE) :WINTER - 2017

SUBJECT : MATHEMATICS: CALCULUS OF SERVERAL VARIABLES (M - 31)

Day : Monday
Date : 06/11/2017

W-2017-0621

Time : 12.00 NOON TO 02.00 PM
Max. Marks : 40

N. B. ;

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable calculator is **ALLOWED**.

Q. 1 Attempt any **TWO** of the following: **(10)**

a) Show that if a function $f(x, y)$ is differentiable at (a, b) then ***(i)*** the partial derivatives $f_x(a, b)$ and $f_y(a, b)$ exist ***(ii)*** f is continuous at (a, b).

b) If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$ then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$

and $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x\,\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = 2\sin u \cos 3u$.

c) If $u = f(x, y)$ *and* $x = r\cos\theta,\ y = r\sin\theta$

then show that $\left(\dfrac{\partial u}{\partial x}\right)^2 + \left(\dfrac{\partial u}{\partial y}\right)^2 = \left(\dfrac{\partial u}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial u}{\partial \theta}\right)^2$.

Q. 2 Attempt any **TWO** of the following: **(10)**

a) State and prove Taylor's theorem for a function of two variables x and y.

b) Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ by using Taylor's theorem.

c) Investigate the maximum and minimum values of

$f(x, y) = (x + y - 1)(x^2 + y^2)$.

Q. 3 Attempt any **TWO** of the following: **(10)**

a) Change the order of integration and hence evaluate $\displaystyle\int_0^a\left[\int_y^a \frac{y\,dx}{\sqrt{x^2 + y^2}}\right]dy$.

b) Find the area between the curves $x^2 = 4y$ *and* $x^2 = 8 - 4y$.

c) Evaluate $\displaystyle\iint_D (y - x)\,dx\,dy$ over the region D in the xy-plane bounded by the lines $y = x + 1,\ y = x - 3,\ y = -\dfrac{1}{3}x + \dfrac{7}{3}$ and $y = -\dfrac{1}{3}x + 5$.

P. T. O.

Q. 4 Attempt any **FIVE** of the following: **(10)**

a) Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4}{x^4 + y^4 - x}$ along the path $y^4 = x$.

b) If $u = x^3 \sin y$ then find $\dfrac{\partial^3 u}{\partial y \partial x^2}$.

c) If $u = \sin^{-1}\left(\dfrac{x + y}{\sqrt{x} + \sqrt{y}}\right)$, find $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$.

d) If $u = x - y + z,\ v = x^2 + y^2 - z^2,\ w = x^3 - y,$

then evaluate $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$.

e) If $z = f(u, v)$, where $u = x + y,\ v = x - y,$

then show that $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 2\dfrac{\partial z}{\partial u}$.

f) Change the order of integration of $\int_0^2 \left[\int_{2x}^{6-x} f\, dy\right] dx$.

g) Evaluate $\iint_R xy\,(x+y)\,dxdy$, where R is the rectangle $0 \le x \le 1,\ 0 \le y \le 1$.

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