F.Y.B.SC. SEM – II (CBCS - 2016 COURSE) : WINTER - 2017
SUBJECT : MATHEMATICS : INTEGRAL CALCULUS & DIFFERENTIAL EQUATIONS

Day : **Thursday** Time : **03.00 PM TO 06.00 PM**
Date : **09/11/2017** Max. Marks : 60

W-2017-0567

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.

Q.1 A) Select the correct alternatives of the following: **[06]**

i) $\int_0^{\pi/2} \cos^7 \theta \, d\theta =$ __________.

a) $\frac{32}{105}$ **b)** $\frac{64}{105}$ **c)** $\frac{8}{15}$ **d)** $\frac{48}{105}$

ii) $\int_0^{\pi/2} \sin^4 \theta \cos^2 \theta \, d\theta =$ __________.

a) $\frac{5\pi}{128}$ **b)** $\frac{\pi}{64}$ **c)** $\frac{\pi}{32}$ **d)** $\frac{\pi}{16}$

iii) $\int \sqrt{a^2 - x^2} \, dx =$ _______.

a) $\sin^{-1}\left(\frac{x}{a}\right)$

b) $\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + c$

c) $\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \log\left[x + \sqrt{a^2 - x^2}\right]$

d) None of these

iv) Degree of the differential equation $2y = \frac{ax}{\left(\frac{dy}{dx}\right)} + x\frac{dy}{dx}$ is _______.

a) 2 **b)** 3 **c)** 1 **d)** None of these

v) If in the differential equation $Mdx + Ndy = 0$, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ then the differential equation is _______.

a) exact **b)** linear **c)** non-linear **d)** homogeneous

vi) Integrating factor of the differential equation $Mdx + Ndy = 0,\ f_1(xy)y\,dx + f_2(xy)x\,dy = 0,$ is _______.

a) $\frac{1}{Nx + My}$ if $Nx + My \neq 0$ **b)** $\frac{1}{Nx - My}$ if $Nx - My \neq 0$

c) $\frac{1}{Mx - Ny}$ if $Mx - Ny \neq 0$ **d)** $\frac{1}{Mx + Ny}$ if $Mx + Ny \neq 0$

P.T.O.

B) Solve the following: **[06]**

i) State the formula for obtaining surface area of the curve $r = f(\theta)$.

ii) For evaluating $\int \frac{dx}{4+5\cos x}$, which is the substitution?

iii) Evaluate : $\int_0^{\pi/2} \sin^4 x \cos^4 x \, dx$.

iv) Define linear differential equation.

v) Define integrating factor of the differential equation.

vi) Form the differential equation of $y = ae^{-2x} + be^{2x}$, where a and b are arbitrary constants.

Q.2 Attempt **ANY THREE** of the following: **[12]**

a) Show that $\int_0^{\pi/2} \sin^n x \, dx = \frac{n-1}{n} \int_0^{\pi/2} \sin^{n-2} x \, dx$.

b) Evaluate : $\int \frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} dx$.

c) Evaluate : $\int \frac{x^2+2}{x^3-1} dx$.

d) Find the orthogonal trajectories of the family of rectangular hyperbolas $xy = c^2$.

Q.3 Attempt **ANY FOUR** of the following: **[12]**

a) Define homogeneous differential equation and explain the method of its solution.

b) Solve the differential equation $\frac{dy}{dx} = \frac{x+2y+1}{2x+4y+3}$.

c) Solve the differential equation $(1+xy^2)dx + (1+x^2y)\,dy = 0$.

d) Evaluate : $\int_0^{\pi/2} \sin^6 3x \, dx$.

e) Evaluate : $\int \tan^4 x \, dx$

Q.4 Attempt **ANY TWO** of the following: **[12]**

a) Prove that the solution of the differential equation of the form $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x alone is $y e^{\int P dx} = \int e^{\int P dx} . Q \, dx + C$.

b) Solve : $y(xy+1)\,dx + x(1+xy+x^2y^2)\,dy = 0$.

c) Solve : $\frac{dy}{dx} + \frac{4xy}{x^2+1} = \frac{1}{(x^2+1)^3}$.

Q.5 Attempt **ANY TWO** of the following: **[12]**

a) Evaluate : $\int \frac{x^2 \, dx}{x^4+1}$.

b) Find the length of the arc of the curve $x = 1 - \cos t + \frac{3}{5}t$, $y = \frac{4}{5}\sin t$ between t = 0 and t = π.

c) Find the area of the surface of revolution generated by revolving about the y – axis, the curve $x = y^3$ from $y = 0$ to $y = 2$.

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