S.Y. B Sc – III (CBCS – 2016 COURSE) : WINTER – 2017

SUBJECT: MATHEMATICS: GROUP THEORY & DIFFERENTIAL EQUATIONS

Day : Friday Time 11.00 A.M. TO 02.00 PM
Date : 10/11/2017 Max. Marks :60

W-2017-0580

N. B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt any **TWO** of the following: **(12)**

a) If a, b are any two elements in a group G then show that $O(a) = O(b^{-1}ab)$.

b) Let $S = \{1, -1, i, -i\}$. Show that $(S, \bullet)$ is an abelian group where $\bullet$ is usual multiplication of complex numbers.

c) Prove that group having four elements must be abelian.

Q.2 Attempt any **TWO** of the following: **(12)**

a) Prove that a non-empty subset H of G is subgroup of G if and only if the following two conditions are satisfied.
$i)\, a, b \in H \Rightarrow ab \in H$ $ii)\, a \in H \Rightarrow a^{-1} \in H.$

b) Let G be the group of all 2×2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $ad - bc \neq 0$ under matrix multiplication.
$Let\ H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} / \ ad \neq 0 \right\}$ show that H is a subgroup of G.

c) A and B are subgroups of a group G such that $A \cup B$ is also a subgroup of G. Show that $A \subseteq B \ or \ B \subseteq A$.

Q.3 Attempt any **TWO** of the following: **(12)**

a) Show that if $f(D)y = e^{ax}\,\mathrm{V}$, where V is a function of x
then $\frac{1}{f(D)}\left(e^{ax}V\right) = e^{ax}\frac{1}{f(D+a)}\mathrm{V}$.

b) Solve : $\left(D^4 + 4\right)y = \cos 2x + \cos 4x + x$.

c) Solve : $\left(D^2 - 6D + 13\right)y = e^{3x}\sin 2x$.

P. T. O.

Q.4 Attempt any **THREE** of the following: **(12)**

a) Show that by using the substitutions $x^2 = u$ and $y^2 = v$, differential equation $(px - y)(py + x) = 2p$ becomes Clairaut's equation and hence solve it.

b) Solve : $y = 2px + x^2 p^4$, where $p = \frac{dy}{dx}$.

c) Solve : $\left(\frac{dy}{dx}\right)^2 - 5\left(\frac{dy}{dx}\right) + 6 = 0$.

d) Solve : $y = 2px - p^2$, where $p = \frac{dy}{dx}$.

Q.5 Attempt any **FOUR** of the following: **(12)**

a) Show that $(Z_5, +_5)$ is cyclic group. Find all its generators.

b) Consider the group $(S, \bullet)$ where $S = \{1, \omega, \omega^2\}$ and $\bullet$ is usual multiplication of complex numbers. Find inverse of every element of S.

c) Define: i) Group ii) Semi- group iii) cyclic group

d) Find the complementary solution of $(D^2 + 2D + 5)y = x \sin 2x$.

e) Solve : *i)* $y = px + \sqrt{a^2 p^2 + b^2}$ *ii)* $(y - px)^2 = 1 + p^2$

iii) $y - 2px = f(p^2)$

f) Find the particular integral of $(D^2 + 4)y = x.\sin x$.

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