## F.Y.B.SC. SEM – I (CBCS - 2016 COURSE) : WINTER - 2017
### SUBJECT : MATHEMATICS : CALCULUS

Day : **Saturday** Time : **11.00 A.M. TO 02.00 PM**
Date : **11/11/2017** **W-2017-0553** Max. Marks : 60

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**N.B.**

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.

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**Q.1 A)** Choose the correct alternatives of the following: **(06)**

**i)** If $y=\dfrac{1}{ax+b}$ then $y_n=....$

**a)** $\dfrac{(-1)^n n!a^n}{(ax+b)^{n+1}}$ **b)** $\dfrac{(-1)^n n!a^n}{(ax+b)^n}$ **c)** $\dfrac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$ **d)** None of these

**ii)** If $y=\sin^3 x$ then $y_n=....$

**a)** $\dfrac{3^n}{4}\sin\left(3x+n\dfrac{\pi}{2}\right)$ **b)** $\dfrac{3^n}{4}\sin\left(x+\dfrac{n\pi}{2}\right)-\dfrac{3^n}{4}\sin\left(3x+\dfrac{n\pi}{2}\right)$

**c)** $\dfrac{3}{4}\cos\left(x+\dfrac{n\pi}{2}\right)-\dfrac{3^n}{4}\cos\left(3n+\dfrac{n\pi}{2}\right)$ **d)** $\dfrac{3}{4}\sin\left(x+\dfrac{n\pi}{2}\right)-3^n\sin\left(3x+(n-1)\dfrac{\pi}{2}\right)$

**iii)** A series $\sum_{n=1}^{\infty}\dfrac{3n+5}{(n+1)(n+2)}$ is

**a)** divergent **b)** oscillatory **c)** convergent **d)** none of these

**iv)** $\lim_{x\to\frac{\pi}{2}} \dfrac{\log\left(x-\dfrac{\pi}{2}\right)}{\tan x}=.....$

**a)** $\dfrac{1}{2}$ **b)** $\dfrac{3}{2}$ **c)** $-1$ **d)** $0$

**v)** A sequence $\{a_n\}$ where $a_n=2(-1)^n-\dfrac{3}{n}$ is

**a)** convergent **b)** divergent **c)** oscillatory **d)** none of these

**vi)** $1+x+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+...+\dfrac{x^n}{n!}=....,\forall x\in R,$ is an expansion of

**a)** $\dfrac{1}{x}$ **b)** $e^x$ **c)** $\log x$ **d)** none of these

**B)** Answer the following: **(06)**

**i)** Define supremum of a function..

**ii)** State Heine's property.

**iii)** If $y=e^{ax}\sin(bx+c)$, then find $y_n$.

**iv)** Evaluate $\lim_{x\to 0}\dfrac{xe^x-\log(1+x)}{x^2}$

**v)** State geometrical meaning of Lagrange's mean value theorem.

**vi)** State Maclaurin's power series of $f(x)$ .

**Q.2** Attempt any **THREE** of the following: **(12)**

**a)** State and prove Rolle's mean value theorem.

**b)** Use Cuachy's mean value theorem to obtain value of C for the functions $f(x) = e^x$ and $g(x) = x$ over [0, 1]

**c)** Verify Lagrange's mean value theorem for the function $f(x) = 2x^2 - 7x + 10$ over $[2, 5]$ . find the value of C and $\theta$ .

**d)** Show that $\{a_n\}$ where $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + ... + \frac{1}{2n}$ is bounded.

**Q.3** Attempt any **FOUR** of the following: **(12)**

**a)** Prove that every continuous function on closed and bounded interval attains its bounds.

**b)** If the functions defined below is continuous is given domain, find the value of $a$ and $b$, where

$$f(x) = ax + b, \text{ for } 0 \le x < 2$$
$$= bx + 11, \text{ for } 2 \le x \le 4; \text{ and } f(3) = 2$$

**c)** Discuss the continuity of $f(x) = \sqrt{\frac{x+1}{x-4}}$

**d)** Evaluate $\lim_{x \to 4} \left[ \frac{1}{\log(x-3)} - \frac{1}{x-4} \right]$

**e)** Verify Rolle's theorem for the function $f(x) = e^x(\sin x - \cos x)$, over $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

**Q.4** Attempt any **TWO** of the following: **(12)**

**a)** Show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ , is convergent if $p > 1$ .

**b)** Show that a sequence $\{a_n\}$ is monotonic and bounded, where

$$a_n = \frac{1}{3+1} + \frac{1}{3^2+1} + \frac{1}{3^3+1} + .... + \frac{1}{3^n+1}$$

**c)** Discuss the convergence of the series

$$= \frac{x}{1 \cdot 2 \cdot 3} + \frac{3x^2}{2 \cdot 3 \cdot 4} + \frac{5x^3}{3 \cdot 4 \cdot 5} + \frac{7x^4}{4 \cdot 5 \cdot 6} + ...$$

**Q.5** Attempt any **TWO** of the following: **(12)**

**a)** State and prove Leibnitz's theorem for $n^{th}$ derivative of the product of two functions of $x$.

**b)** If $y = \sin\left(m \sin^{-1} x\right)$, then show that

$$(1 - x^2) y_{n+2} - (2n+1) x\, y_{n+1} + (m^2 - n^2) y_n = 0$$

**c)** By using Taylor's theorem, prove that

$$e^x \cos x = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} - \frac{x^5}{30} + ...$$

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