F.Y.B.SC. SEM – I (2014 COURSE) : WINTER - 2017

SUBJECT : MATHEMATICS : CALCULUS (M-12)

Day : **Thursday** Time **12.00 NOON TO 02.00 PM**

Date : **09/11/2017** Max. Marks : 40

W-2017-0597

N.B.

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt any **TWO** of the following: **(10)**

a) Show that if f is continuous on $[a,b]$ and $f(a)$ and $f(b)$ have opposite signs, then $f(x)=0$, for some $x\in[a,b]$.

b) Discuss the continuity of function f if

$f(x)=\frac{e^{1/x}-1}{e^{1/x}+1}$, when $x\neq 0$ and $f(0)=0$.

c) If $y=a\cos(\log x)+b\sin(\log x)$, then show that

$x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$.

Q.2 Attempt any **TWO** of the following: **(10)**

a) State and prove Rolle's mean value theorem.

b) For $0<a<b$, prove by using Lagrange's mean value theorem that

$1-\frac{a}{b}<\log\left(\frac{b}{a}\right)<\frac{b}{a}-a$ and hence show that $\frac{1}{6}<\log\left(\frac{6}{5}\right)<\frac{1}{5}$.

c) If $f'(x)$ exists in the interval (a,b) where $0<a<b$, then prove that there exists a number $c\in(a,b)$ such that $f(b)-f(a)=cf'(c)\log\left(\frac{b}{a}\right)$.

Q.3 Attempt any **TWO** of the following: **(10)**

a) Show that a sequence $\{S_n\}$ where $S_n=\left(1+\frac{1}{n}\right)^n$ is monotonic and bounded.

b) Discuss the convergence of $\sum_{n=1}^{\infty}\frac{(n-1)x^n}{(n-3)(n-4)}$ is convergent if $p>1$.

c) Using Maclaurin's series expansion, prove that

$e^x\cos x=1+x-\frac{x^3}{3}-\frac{x^4}{6}-\frac{x^5}{30}+..$

Q.4 Attempt any **FIVE** of the following: **(10)**

a) Show that every differentiable function is continuous.

b) State Leibnitz's theorem for n^{th} derivative.

c) Evaluate $\lim_{x\to 0}\left[\frac{1}{x}-\frac{1}{\sin x}\right]$.

d) By using Maclaurins theorem find the expansion of e^x.

e) If $y=\frac{1}{4x+5}+e^{7x}+\log(3x-2)+2^{5x}$, find y_n.

f) Discuss the convergence of sequence $\{a_n\}$ where $a_n=(-1)^n(3n+4)$.

g) Discuss the convergence of $\sum_{n=1}^{\infty}\frac{7-3n}{5n^2+3n+8}$.

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