# F.Y.B.SC. SEM – I (2014 COURSE) : WINTER - 2017

## SUBJECT : MATHEMATICS : ALGEBRA (M – 11)

Day : **Tuesday**
Date : **07/11/2017**

**W-2017-0595**

Time : **12.00 NOON TO 02.00 PM**
Max. Marks : 40

**N.B.:**

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

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**Q.1** Attempt **ANY TWO** of the following: **[10]**

**a)** Prove that if A is a square matrix of order n, then the matrices A and adj A. commute and the product is the scalar matrix |A| I.

**b)** Determine the values of x (if any) that will make the matrix A given below of **i)** rank 1 **ii)** rank 2 **iii)** rank 3.

$$A = \begin{bmatrix} x & x & 2 \\ 2 & x & x \\ x & 2 & x \end{bmatrix}.$$

**c)** Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

**Q.2** Attempt **ANY TWO** of the following: **[10]**

**a)** Show that if a, b $\in$ Z and $b > 0$, then there exists unique integers $q$ and $r$ such that $a = bq + r$, where $0 \le r < b$.

**b)** Show that the integers 1357 and 1166 are relatively prime. Find integers $m$ and $n$ such that $1 = 1357m + 1166n$.

**c)** Investigate for what values of $\lambda$ and $\mu$ the simultaneous equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have (i) no solution (ii) unique solution (iii) an infinite number of solutions.

**Q.3** Attempt **ANY TWO** of the following: **[10]**

**a)** State De Moivre's theorem and prove it for positive and negative integers.

**b)** Find the value of $\left(1+i\sqrt{3}\right)^{10} + \left(1-i\sqrt{3}\right)^{10}$.

**c)** Find the cube roots and fourth roots of unity.

**Q.4** Attempt **ANY FIVE** of the following: **[10]**

**a)** Write in the form x + iy of $\dfrac{(\cos\theta + i\sin\theta)^3}{(\cos\theta - i\sin\theta)^2}$.

**b)** Find the modulus and argument of $z = \dfrac{3-i}{2+i} + \dfrac{3+i}{2-i}$.

**c)** If (a, b) = 1 and a | c, b | c then show that ab | c.

**d)** Define: **i)** Relatively prime integers **ii)** Greatest common divisor.

**e)** Show that if A is a non-singular matrix then $\left|A^{-1}\right| = \dfrac{1}{|A|}$.

**f)** Find rank A if $A = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 5 & 4 \\ 3 & 9 & 7 \end{bmatrix}$.

**g)** Find $A^{-1}$, if $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$.

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