S.Y.B.SC. SEM – III (CBCS - 2016 COURSE) : WINTER - 2017
SUBJECT : MATHEMATICS: CALCULUS OF SEVERAL VARIABLES

Day : **Wednesday** Time : **11.00 A.M. TO 02.00 PM**
Date : **08/11/2017** **W-2017-0578** Max. Marks :60

N. B.:

1) All questions are **COMPULOSRY.**
2) Figures to the **RIGHT** indicate full marks.

Q.1 Attempt any **TWO** of the following: **(12)**

a) Show that if a function f(x, y) is differentiable at (a, b), then (i) the partial derivatives f_x (a,b) and f_y (a, b) exist and (ii) f is continuous at (a, b).

b) Prove that $u=\frac{1}{\sqrt{x^2+y^2+z^2}}\left(x^2+y^2+z^2\neq 0\right)$, satisfies the partial differential equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$.

c) If x = rcosθ, y = rsinθ, show that

$$\frac{\partial^2\theta}{\partial x\partial y}=\frac{\partial^2(\log r)}{\partial x^2}=-\frac{\partial^2(logr)}{\partial y^2}=-\frac{1}{r^2}\cos 2\theta.$$

Q.2 Attempt any **TWO** of the following: **(12)**

a) Explain Lagrange's method of undetermined multipliers.

b) If $u=\sin^{-1}\left(x^2+y^2\right)^{\frac{1}{5}}$ then prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{2}{25}\tan u\left[2\tan^2 u-3\right].$$

c) Find maxima or minima of $F(x,y) = x^4+y^4 - (x+y)^2$.

Q.3 Attempt any **TWO** of the following: **(12)**

a) State and prove Taylor's theorem for a function of two variables x and y.

b) Using Maclaurin's theorem show that

$$\sin x\sin y=xy-\frac{1}{6}\left[\left(x^3+3xy^2\right)\cos\theta x\sin\theta y+\left(y^3+3x^2y\right)\sin\theta x\cos\theta y\right],$$

$0<\theta<1$.

c) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

P.T.O.

Q.4 Attempt any **THREE** of the following: **(12)**

a) Find the volume of the solid in the first octant bounded by two cylinders $x^2 + y^2 = 4$ and $x^2 + z^2 = 4$.

b) Find the area between the curves $y = x^2$ and $y = 4x - x^2$.

c) Change the order of integration and hence evaluate $\int_0^1 \left[\int_y^1 e^{-x^2} dx \right] dy$.

d) Evaluate $\iint_D (y-x)\, dxdy$, over the region D in the xy-plane bounded by the lines $y = x+1, y = x-3, y = -\frac{1}{3}x + \frac{7}{3}, \ y = -\frac{1}{3}x + 5$.

Q.5 Attempt any **FOUR** of the following: **(12)**

a) Prove that if f has an extremum at a point (a,b) and if the partial derivatives $f_x(x,y)$ and $f_y(x,y)$ exist in a neighborhood of (a,b) then $f_x(a,b) = 0 = f_y(a,b)$.

b) Can f(0,0) be defined so that f(x,y) is continuous at (0,0) if $f(x,y) = \frac{\sin(x^2 + y)}{x + y}$.

c) Show that u is harmonic function if $u = \log(x^2 + y^2)$.

d) If $u = \frac{x+y}{1-xy}, v = \tan^{-1} x + \tan^{-1} y$ then find $\frac{\partial(u,v)}{\partial(x,y)}$.

e) Evaluate $\iint_R (x \sin y - ye^x)\, dxdy$, where R is the rectangle $-1 \le x \le 1, \ 0 \le y \le \frac{\pi}{2}$.

f) Change the order of integration $\int_{-1}^{1} \left[\int_{-\sqrt{1-x^2}}^{1-x^2} f\, dy \right] dx$.

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