F.Y.B.SC. SEM – I (CBCS - 2016 COURSE) : WINTER - 2017
SUBJECT : MATHEMATICS : ALGEBRA

Day : Thursday
Date : 09/11/2017

Time : **11.00 A.M. TO 02.00 PM**
Max. Marks : 60

W-2017-0551

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 A) Select the correct alternatives of the following: **[06]**

i) If $A = \begin{bmatrix} 3 & 5 \\ 1 & 3 \end{bmatrix}$, then A^{-1} = ______.

a) $\frac{1}{4}\begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}$ **b)** $\frac{1}{4}\begin{bmatrix} 3 & -5 \\ -1 & 3 \end{bmatrix}$

c) $\frac{1}{4}\begin{bmatrix} 3 & 5 \\ 1 & 3 \end{bmatrix}$ **d)** $-\frac{1}{4}\begin{bmatrix} 3 & -5 \\ -1 & 3 \end{bmatrix}$

ii) Rank of matrix $\begin{bmatrix} 2 & -3 & 4 \\ 1 & 1 & -1 \\ 3 & -2 & 3 \end{bmatrix}$, is _______.

a) 1 **b)** 3 **c)** 2 **d)** None of these

iii) If z = 2 + 3i then $z\bar{z}$ = _______.

a) 4 **b)** 5 **c)** −5 **d)** 13

iv) Let a and b be non-zero integers and let $d = (a, b)$. If $a = xd$ and $b = yd$ then (x, y) = _______.

a) 1 **b)** b **c)** a **d)** d

v) Arg $\left(\frac{2+i}{2-i}\right)$ is _________.

a) $\tan^{-1}\left(\frac{1}{2}\right)$ **b)** $\tan^{-1} 2$ **c)** $\frac{\pi}{4}$ **d)** $\tan^{-1}\left(\frac{4}{3}\right)$

vi) If $(a, b) = d$ then $\left(\frac{a}{d}, \frac{b}{d}\right)$ = _______.

a) d **b)** 1 **c)** ad **d)** bd

B) Answer the following questions: **[06]**

i) Define adjoint of a square matrix.

ii) Find the characteristic equation of $\begin{bmatrix} 2 & 6 \\ 0 & 3 \end{bmatrix}$

iii) If A is a non-singular matrix then show that $(A')^{-1} = (A^{-1})'$.

iv) State fourth roots of unity.

v) Show that sum and product of complex numbers conjugate to each other is real.

vi) Define congruence relation.

P.T.O.

Q.2 Attempt **ANY THREE** of the following: **[12]**

a) Prove that if p is prime and a and b are integers such that p|ab then either p|a or p|b.

b) Find the eigne values of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 2 & 3 \end{bmatrix}$.

c) Test the following equations for consistency. If they are consistent find their general solution:
$2x - y - z = 4,\ x - 2y + 4z = -1,\ 3x - 4y + 6z = 1$.

d) Find the expression of $\cos^5\theta$ in terms of cosine and sine of multiples of θ.

Q.3 Attempt **ANY FOUR** of the following: **[12]**

a) If z_1 and z_2 are any two complex numbers and $|z_2| \neq 0$, then show that

i) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ **ii)** $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$.

b) Prove that $\left(1+i\sqrt{3}\right)^{-10} = 2^{-11}\left(-1+i\sqrt{3}\right)$.

c) Find five-fifth roots of -1.

d) Find the value of $\left(1+i\sqrt{3}\right)^5 + \left(1-i\sqrt{3}\right)^5$.

e) Find the cube roots of unity.

Q.4 Attempt **ANY TWO** of the following: **[12]**

a) Prove that if A is a square matrix of order n, then the matrices A and adj A commute and the product is the scalar matrix |A|I i.e., A(adjA)=(adjA)A= |A|I.

b) Find the non-singular matrices P and Q such that PAQ is normal form and find rank A, where $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$.

c) Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

Q.5 Attempt **ANY TWO** of the following: **[12]**

a) Prove that if a and b are any two integers with $b \neq 0$, then there exist unique integers q and r such that $a = bq + r$, where $0 \leq r \leq |b|$

b) Find g.c.d. 'd' of 3997 and 2947 and express it in the form d =3997m + 2947n for some $m,\ n \in z$.

c) Write the system of equations
$ax + by + cz = 0,\ bx + cy + az = 0,\ cx + ay + bz = 0$ as a matrix equation. Show that the equations have a non-trivial solution if and only if
$a + b + c = 0$ or $a = b = c$.

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